Chapter 8 The Facet Model

8.1 Introduction

facet model: image as continuum or piecewise continuous intensity surface
observed digital image: noisy, discretized sampling of distorted version
general forms:

1. piecewise constant (flat facet model), ideal region: constant gray level
2. piecewise linear (sloped facet model), ideal region: sloped plane gray level
3. piecewise quadratic, gray level surface: bivariate quadratic
4. piecewise cubic, gray level surface: cubic surfaces

8.2 Relative Maxima

relative maxima: first derivative zero, second derivative negative

8.3 Sloped Facet Parameter and Error Estimation

least-squares procedure: to estimate sloped facet parameter, noise variance

8.4 Facet-Based Peak Noise Removal

peak noise pixel: gray level intensity significantly differs from neighbors
(a) peak noise pixel, (b) not
=====Fig. 8.1=====

8.5 Iterated Facet Model

facets: image spatial domain partitioned into connected regions
facets: satisfy certain gray level and shape constraints
facets: gray levels as polynomial function of row-column coordinates

8.6 Gradient-Based Facet Edge Detection

gradient-based facet edge detection: high values in first partial derivative

8.7 Bayesian Approach to Gradient Edge Detection

The Bayesian approach to the decision of whether or not an observed gradient magnitude G is statistically significant and therefore participates in some edge is to decide there is an edge (statistically significant gradient) when,

$$P(\text{edge}|G) > P(\text{nonedge}|G)$$

$P(\text{edge}|G)$: given gradient magnitude, conditional probability of edge
$P(\text{nonedge}|G)$: given gradient magnitude, conditional probability of nonedge

$$P(\text{edge}|G) = \frac{P(G|\text{edge})P(\text{edge})}{P(G)}$$

$$P(\text{nonedge}|G) = \frac{P(G|\text{nonedge})P(\text{nonedge})}{P(G)}$$

$$P(G|\text{edge})P(\text{edge}) > P(G|\text{nonedge})P(\text{nonedge})$$

possible to infer $P(G|\text{edge})$ from observed image data

$$P(G) = P(G|\text{edge})P(\text{edge}) + P(G|\text{nonedge})P(\text{nonedge})$$

$$\begin{aligned} P(G|\text{edge}) &= \frac{P(G) - P(G|\text{nonedge})P(\text{nonedge})}{P(\text{edge})} \\ &= \frac{P(G) - P(G|\text{nonedge})P(\text{nonedge})}{1 - P(\text{nonedge})} \end{aligned}$$

8.8 Zero-Crossing Edge Detector

gradient edge detector: looks for high values of first derivatives
zero-crossing edge detector: looks for relative maxima in first derivative
zero-crossing: pixel as edge if zero crossing of second directional derivative
underlying gray level intensity function f takes the form

$$f(r,c) = k_1 + k_2 r + k_3 c + k_4 r^2 + k_5 rc + k_6 c^2 + k_7 r^3 + k_8 r^2 c + k_9 rc^2 + k_{10} c^3$$

8.8.1 Discrete Orthogonal Polynomials

discrete orthogonal polynomial basis set of size N: polynomials deg. $0..N-1$
discrete Chebyshev polynomials: these unique polynomials

$$P_0(r) = 1$$

$$P_1(r) = r$$

$$P_n(r) = r^n + a_{n-1} r^{n-1} + \ldots + a_1 r + a_0$$

$P_n(r)$: orthogonal to $P_0(r), \ldots, P_{n-1}(r)$, thus

$$\sum_{r \in R} P_k(r)(r^n + a_{n-1} r^{n-1} + \ldots + a_1 r + a_0) = 0, \quad k = 0, \ldots, n-1$$

discrete orthogonal polynomials can be recursively generated

$$P_{i+1}(r) = rP_i(r) - \beta_i P_{i-1}(r)$$

$$\beta_i = \frac{\sum_{r \in R} rP_i(r)P_{i-1}(r)}{\sum_{r \in R} P_{i-1}(r)^2}$$

$$P_2(r) = r^2 - \frac{\mu_2}{\mu_0}$$

$$P_3(r) = r^3 - \frac{\mu_4}{\mu_2} r$$

$$P_4(r) = r^4 + \frac{(\mu_2\mu_4 - \mu_0\mu_6)r^2 + (\mu_2\mu_6 - \mu_4^2)}{\mu_0\mu_4 - \mu_2^2}$$

$$\mu_k = \sum_{s\in R} s^k$$

8.8.2 Two-Dimensional Discrete Orthogonal Polynomials
2-D discrete orthogonal polynomials creatable from tensor products of 1D from above equations

Index Set	Discrete Orthogonal Polynomial Set
$\{-\frac{1}{2}, \frac{1}{2}\}$	$\{1, r\}$
$\{-1, 0, 1\}$	$\{1, r, r^2 - \frac{2}{3}\}$
$\{-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}\}$	$\{1, r, r^2 - \frac{5}{4}, r^3 - \frac{41}{20}r\}$
$\{-2, -1, 0, 1, 2\}$	$\{1, r, r^2 - 2, r^3 - \frac{17}{5}, r^4 + 3r^2 + \frac{72}{35}\}$

8.8.3 Equal-Weighted Least-Squares Fitting Problem
weight

$$\frac{P_m(r)}{\sum_{s\in R} P_m^2(s)}$$

e.g. r: row, c: column, k_5: rc numerator, $\sum(rc)^2$ denominator
=====Fig. 8.8=====

8.8.4 Directional Derivative Edge Finder
We define the directional derivative edge finder as the operator that places an edge in all pixels having a negatively sloped zero crossing of the second directional derivative taken in the direction of the gradient
r: row
c: column
ρ: radius in polar coordinate
α: angle in polar coordinate, clockwise from column axis
$r = \rho \sin\alpha, c = \rho\cos\alpha$
directional derivative of f at point (r, c) in direction α:

$$f'_\alpha(r, c) = \lim_{h\to 0} \frac{f(r + h\sin\alpha, c + h\cos\alpha) - f(r, c)}{h}$$

$$f'_\alpha(r, c) = \frac{\partial f}{\partial r}(r, c)\sin\alpha + \frac{\partial f}{\partial c}(r, c)\cos\alpha$$

second directional derivative of f at point (r, c) in direction α:

$$f''_\alpha(r, c) = \frac{\partial^2 f}{\partial r^2}(r, c)\sin^2\alpha + 2\frac{\partial^2 f}{\partial r\partial c}(r, c)\sin\alpha\cos\alpha + \frac{\partial^2 f}{\partial c^2}(r, c)\cos^2\alpha$$

$$f'''_\alpha(r, c) = \frac{\partial^3 f}{\partial r^3}(r, c)\sin^3\alpha + 3\frac{\partial^3 f}{\partial r^2\partial c}(r, c)\sin^2\alpha\cos\alpha$$
$$+3\frac{\partial^3 f}{\partial r\partial c^2}(r, c)\sin\alpha\cos^2\alpha + \frac{\partial^3 f}{\partial c^3}(r, c)\cos^3\alpha$$

8.9 Integrated Directional Derivative Gradient Operator

integrated directional derivative gradient operator: more accurate
step edge direction
=====joke=====

8.10 Corner Detection

corners: to detect buildings in aerial images
corner points: to determine displacement vectors from image pair
gray scale corner detectors: detect corners directly by gray scale image

8.11 Isotropic Derivative Magnitudes

gradient edge: from first-order isotropic derivative magnitude

8.12 Ridges and Ravines on Digital Images

A digital ridge (ravine) occurs on a digital image when there is a simply connected sequence of pixels with gray level intensity values that are significantly higher (lower) in the sequence than those neighboring the sequence.
ridges, ravines: from bright, dark lines or reflection, variation ...

8.13 Topographic Primal Sketch

8.13.1 Introduction

The basis of the topographic primal sketch consists of the labeling and grouping of the underlying image-intensity surface patches according to the categories defined by monotonic, gray level, and invariant functions of directional derivatives.
categories:

- peak
- pit
- ridge
- ravine
- saddle
- flat
- hillside

topographic primal sketch: rich, hierarchical, structurally complete rep.

Invariance Requirement

histogram normalization, equal probability quantization: nonlinear, enhancing
peak, pit, ridge, valley, saddle, flat, hillside: have required invariance

Background

primal sketch: rich description of gray level changes present in image
description: includes type, position, orientation, fuzziness of edge
topographic primal sketch: two-dimensional gray level variations

8.13.2 Mathematical Classification of Topographic Structures

topographic structures: invariant under monotonically increasing intensity tran.

Peak

peak: knob: local maximum in all directions
=====Fig. 8.24=====
peak: curvature downward in all directions
at peak: gradient zero
at peak: second directional derivative negative in all directions
point classified as peak if

$$||\nabla f|| = 0, \lambda_1 < 0, \lambda_2 < 0$$

$||\nabla f||$: gradient magnitude
λ_1: second directional derivative in $w^{(1)}$ direction
λ_2: second directional derivative in $w^{(2)}$ direction

Pit

pit: sink: bowl: local minimum in all directions
pit: gradient zero, second directional derivative positive

$$||\nabla f|| = 0, \lambda_1 > 0, \lambda_2 > 0$$

Ridge

ridge: occurs on ridge line
ridge line: a curve consisting of a series of ridge points
walk along ridge line: points to the right and left are lower
ridge line: may be flat, sloped upward, sloped downward, curved upward,...
ridge: local maximum in one direction
=====Fig. 8.25=====

$$||\nabla f|| \neq 0, \lambda_1 < 0, \nabla f \cdot w^{(1)} = 0$$
$$\text{or} \quad ||\nabla f|| \neq 0, \lambda_2 < 0, \nabla f \cdot w^{(2)} = 0$$
$$\text{or} \quad ||\nabla f|| = 0, \lambda_1 < 0, \lambda_2 = 0$$

Ravine

ravine: valley: local minimum in one direction
walk along ravine line: points to the right and left are higher

$$||\nabla f|| \neq 0, \lambda_1 > 0, \nabla f \cdot w^{(1)} = 0$$

$$\text{or} \quad ||\nabla f|| \neq 0, \lambda_2 > 0, \nabla f \cdot w^{(2)} = 0$$

$$\text{or} \quad ||\nabla f|| = 0, \lambda_1 > 0, \lambda_2 = 0$$

Saddle

saddle: local maximum in one direction, local minimum in perpendicular dir.
saddle: positive curvature in one direction, negative in perpendicular dir.
saddle: gradient magnitude zero
saddle: extrema of second directional derivative have opposite signs

$$||\nabla f|| = 0, \lambda_1 * \lambda_2 < 0$$

Flat

flat: plain: simple, horizontal surface
=====Fig. 8.26=====
flat: zero gradient, no curvature

$$||\nabla f|| = 0, \lambda_1 = 0, \lambda_2 = 0$$

flat: foot or shoulder or not qualified at all
foot: flat begins to turn up into a hill
shoulder: flat ending and turning down into a hill
=====joke=====

Hillside

hillside point: anything not covered by previous categories
hillside: nonzero gradient, no strict extrema
slope: tilted flat (constant gradient)

$$\lambda_1 = \lambda_2 = 0$$

convex hill: curvature positive (upward)

$$\lambda_1 \geq \lambda_2 \geq 0, \lambda_1 \neq 0$$

concave hill: curvature negative (downward)

$$\lambda_1 \leq \lambda_2 \leq 0, \lambda_1 \neq 0$$

saddle hill: up in one direction, down in perpendicular direction

$$\lambda_1 * \lambda_2 < 0$$

inflection point: zero crossing of second directional derivative

Summary of the Topographic Categories

mathematical properties of topographic structures on continuous surfaces
=====Table 8.5=====

Invariance of the Topographic Categories

topographic labels: invariant under monotonically increasing gray level tran.
monotonically increasing: positive derivative everywhere

Ridge and Ravine Continua

entire areas of surface: may be classified as all ridge or all ravine

8.13.3 Topographic Classification Algorithm

peak, pit, ridge, ravine, saddle: likely not to occur at pixel center
peak, pit, ridge, ravine, saddle: if within pixel area, carry the label

Case One: No Zero Crossing

no zero crossing along either of two directions: flat or hillside
no zero crossing: if gradient zero, then flat
no zero crossing: if gradient nonzero, then hillside
hillside: possibly inflection point, slope, convex hill, concave hill,...
=====Table 8.6=====

Case Two: One Zero Crossing

one zero crossing: peak, pit, ridge, ravine, or saddle
=====Table 8.7=====

Case Three: Two Zero Crossings

LABEL1, LABEL2: assign label to each zero crossing
=====Table 8.8=====

Case Four: More Than Two Zero Crossing

more than two zero crossings: choose the one closest to pixel center
more than two zero crossings: after ignoring the other, same as case 3

8.13.4 Summary of Topographic Classification Scheme

one pass through the image, at each pixel

1. calculate fitting coefficients, k_1 through k_{10} of cubic polynomial
2. Use above coefficients to find gradient, gradient magnitude, eigenvalues, ...
3. search in eigenvector direction for zero crossing of first derivative
4. recompute gradient, gradient magnitude, second derivative, then classify

=====Garfield 17:8=====

Previous Work
web representation [Hsu et al. 1978]: axes divide image into regions

Project due Dec. 21
Write the following programs to detect edge:
1. zero-crossing on the following four types of images to get edge images (choose proper thresholds), p. 349
2. Laplacian, Fig. 7.33
3. minimum-variance Laplacian, Fig. 7.36
4. Laplacian of Gaussian, Fig. 7.37
5. Difference of Gaussian, (use tk to generate D.O.G.)

D. Marr, *Vision*, W.H. Freeman, San Francisco, p.54-74, 1982.

=====Marr, *Vision*, Fig. 2.9=====

=====Marr, *Vision*, Fig. 2.16=====

dog (inhibitory $\sigma = 1$, excitatory $\sigma = 3$, kernel size=11)