Computer and Robot Vision I

Chapter 3
Binary Machine Vision: Region Analysis

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3.0 Outline

- Part 1 (textbook content):
  - Region properties
  - Signature segmentation properties
- Part 2 Homework

3.1 Introduction

- region: produced by connected components labeling operator

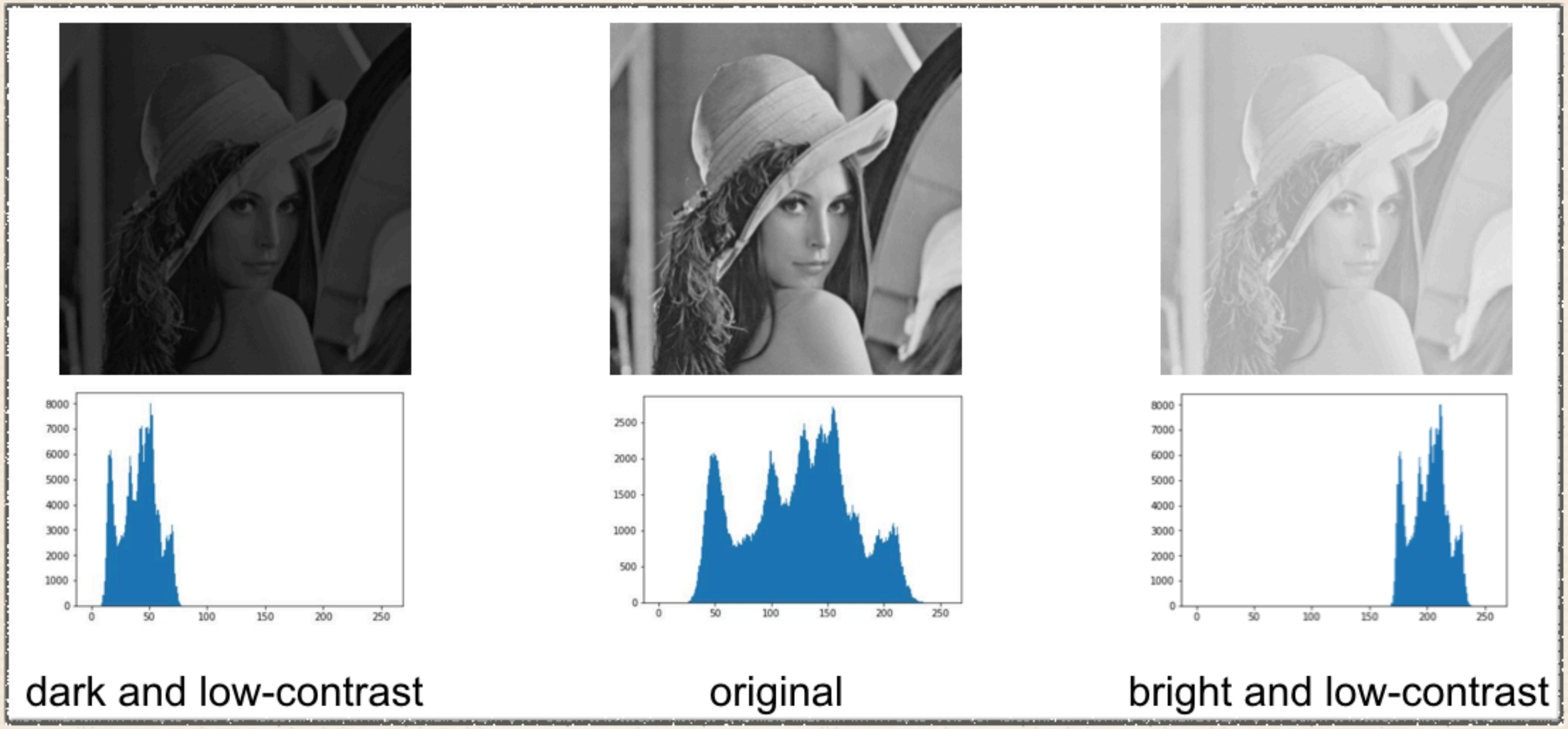
- region properties: measurement vector input to classifier

3.2 Region Properties (cont')

- bounding rectangle: smallest rectangle circumscribes the region
- area: $A = \sum_{(r,c) \in R} 1$
- centroid: $\bar{r} = \frac{1}{A} \sum_{(r,c) \in R} r$ $\bar{c} = \frac{1}{A} \sum_{(r,c) \in R} c$

3.2 Region Properties (cont')

- region intensity histogram:



3.2 Region Properties (cont')

- bounding rectangle: smallest rectangle circumscribes the region

3.2 Region Properties (cont')

- area : A=21

- centroid :

  1. r=3.857

  2. c=(1*2+2*5+3*4+4*4+5*3+6*3)/21=3.476

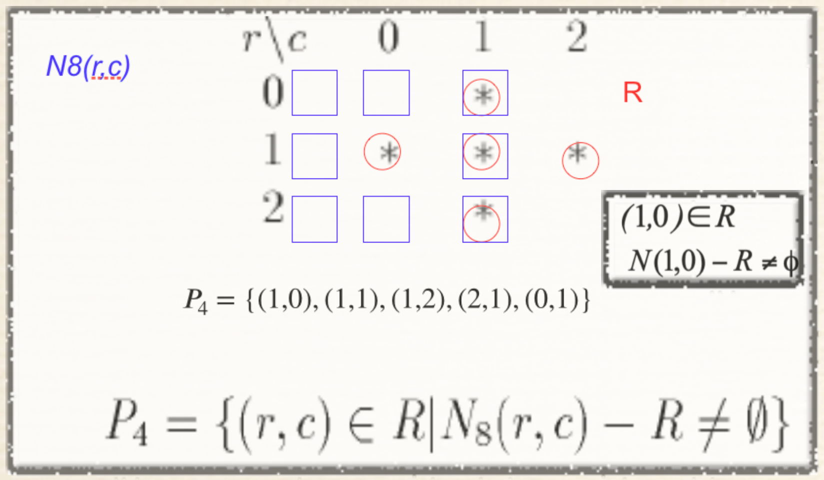
	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	1	0
2	0	0	1	1	1	1	1	0
3	0	0	1	1	1	0	0	0
4	0	0	1	1	1	0	0	0
5	0	1	1	1	1	1	0	0
6	0	1	1	0	0	1	1	0
7	0	0	0	0	0	0	0	0

3.2 Region Properties (cont')

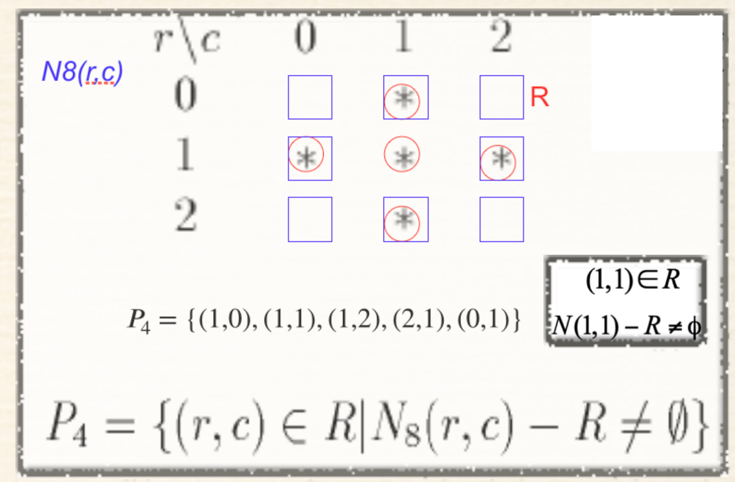
- border pixel: has some neighboring pixel outside the region
- P_4 : 4-connected perimeter: if 8-connectivity for inside and outside
  $$P_4 = \{(r,c) \in R | N_8(r,c) - R \neq \emptyset\}$$
- P_8: 8-connected perimeter: if 4-connectivity for inside and outside
  $$P_8 = \{(r,c) \in R | N_4(r,c) - R \neq \emptyset\}$$

3.2 Region Properties (cont')

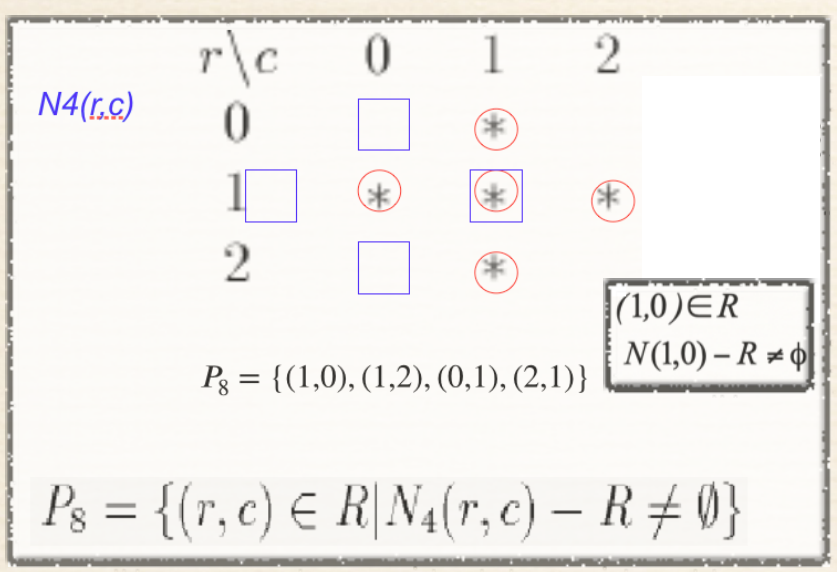
- [x] P4 : if (r,c)=(1,0)



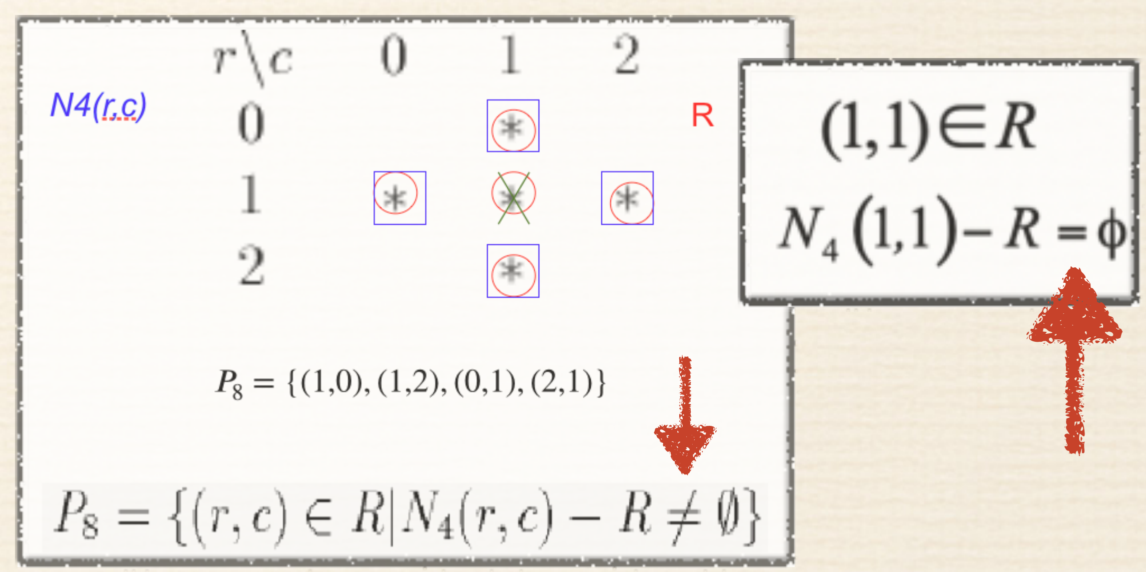
- [x] P4 : if (r,c)=(1,1)



- [x] P8 : if (r,c)=(1,0)



- [ ] P8 : if (r,c)=(1,1)



3.2 Region Properties (cont')

- Eg: center is in P_4 but not in P_8 for

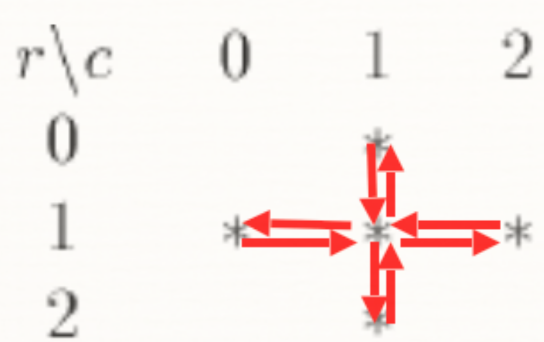
r\c	0	1	2
0		*	
1	*	*	*
2		*	

$P_4 = \{(1,0), (1,1), (1,2), (0,1), (2,1)\}$

$P_8 = \{(1,0), (1,2), (0,1), (2,1)\}$

e.g. $P_4 = \{(1,0) \rightarrow (1,1) \rightarrow (2,1) \rightarrow (1,1) \rightarrow (1,2) \rightarrow (1,1) \rightarrow (0,1) \rightarrow (1,1)\}$

e.g. $P_8 = \{(1,0) \rightarrow (2,1) \rightarrow (1,2) \rightarrow (0,1)\}$



P4

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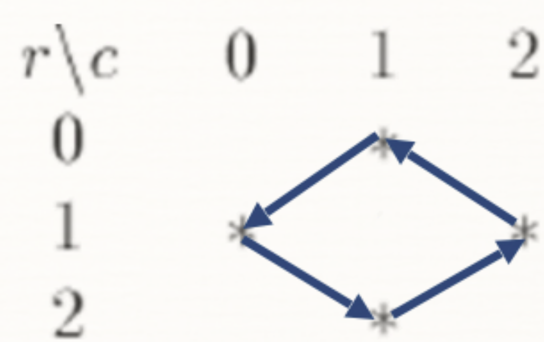


P8

3.2 Region Properties (cont')

- length of perimeter $P =< (r_0, c_0), ..., (r_{K-1}, c_{K-1}) >$,

  successive pixels neighbors

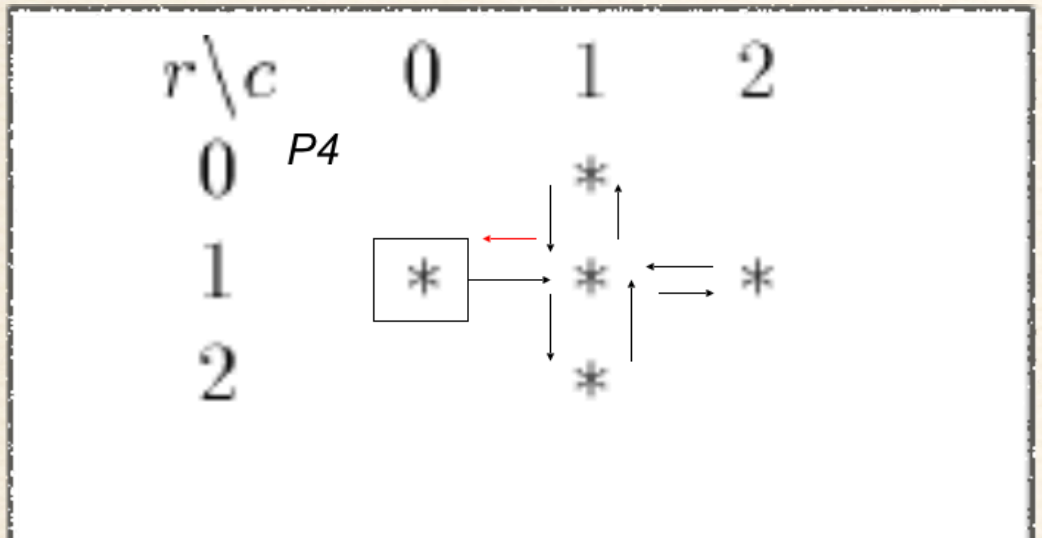
$$|P| = \#\{k | (r_{k+1}, c_{k+1}) \in N_4(r_k, c_k)\}$$

$$+\sqrt{2}\#\{k | (r_{k+1}, c_{k+1}) \in N_8(r_k, c_k) - N_4(r_k, c_k)\}$$

- where *k+1* is computed modulo *K*

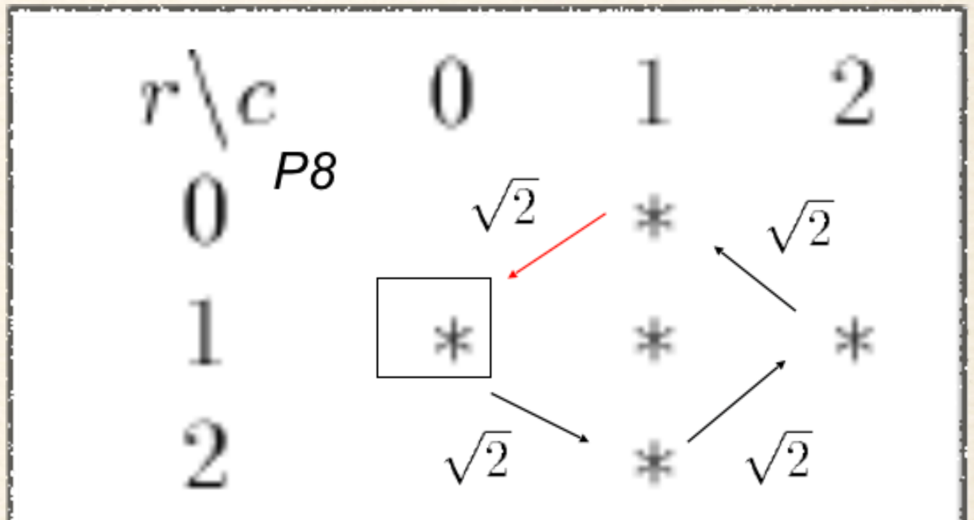
3.2 Region Properties (cont')

- $|P_4| = 8$
  - P_4 : K=8



$P_4 = \langle(1,0), (1,1), (2,1), (1,1), (1,2), (1,1), (0,1), (1,1)\rangle$

- $|P_8| = 4\sqrt{2}$
  - P_8 : K=4



$P_8 = \langle(1,0), (2,1), (1,2), (0,1)\rangle$

3.2 Region Properties (cont')

- mean distance *R* from the centroid to the shape boundary

$$\mu_R = \frac{1}{K} \sum_{k=0}^{K-1} \|(r_k, c_k) - (\bar{r}, \bar{c})\|$$

e.g. $\mu_R(P_4) = 4/8 = 0.5$, $\mu_R(P_8) = 1$

- standard deviation *R* of distances from centroid to boundary

$$\sigma_R^2 = \frac{1}{K} \sum_{k=0}^{K-1} [\|(r_k, c_k) - (\bar{r}, \bar{c})\| - \mu_R]^2$$

e.g. $\sigma_R(P_4) = 0.5$, $\sigma_R(P_8) = 0$

3.2 Region Properties (cont')

$$\mu_R = \frac{1}{K}\sum_{k=0}^{K-1} \|(r_k, c_k) - (\bar{r}, \bar{c})\|$$

- P_4
- P_8

$P_4 = \langle(1,0), (1,1), (2,1), (1,1), (1,2), (1,1), (0,1), (1,1)\rangle$

e.g. $\mu_R(P_4) = 4/8 = 0.5$, $\mu_R(P_8) = 1$

$\mu_R(P_4) = 4/8$

$r \backslash c$	0	1	2
0		$\boxed{*}$	
1	$\boxed{*}$	$\boxed{*}$	$\boxed{*}$
2		$\boxed{*}$	

$(\bar{r}, \bar{c}) = (1,1)$

$\|(1,0) - (\bar{r}, \bar{c})\| = \|(0,-1)\| = 1$

$\|(1,1) - (\bar{r}, \bar{c})\| = \|(0,0)\| = 0$

$\|(2,1) - (\bar{r}, \bar{c})\| = \|(1,0)\| = 1$

$\|(1,1) - (\bar{r}, \bar{c})\| = \|(0,0)\| = 0$

$\|(1,2) - (\bar{r}, \bar{c})\| = \|(0,1)\| = 1$

$\|(1,1) - (\bar{r}, \bar{c})\| = \|(0,0)\| = 0$

$\|(0,1) - (\bar{r}, \bar{c})\| = \|(-1,0)\| = 1$

$\|(1,1) - (\bar{r}, \bar{c})\| = \|(0,0)\| = 0$

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$P_8 = \langle(1,0), (2,1), (1,2), (0,1)\rangle$

e.g. $\mu_R(P_4) = 4/8 = 0.5$, $\mu_R(P_8) = 1$

$\mu_R(P_8) = 1$

$r \backslash c$	0	1	2
0		$\boxed{*}$	
1	$\boxed{*}$	*	$\boxed{*}$
2		$\boxed{*}$	

$(\bar{r}, \bar{c}) = (1,1)$

$\|(1,0) - (\bar{r}, \bar{c})\| = \|(0,-1)\| = 1$

$\|(2,1) - (\bar{r}, \bar{c})\| = \|(1,0)\| = 1$

$\|(1,2) - (\bar{r}, \bar{c})\| = \|(0,1)\| = 1$

$\|(0,1) - (\bar{r}, \bar{c})\| = \|(-1,0)\| = 1$

DC & CV Lab.
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3.2 Region Properties (cont')

$$\sigma_R^2 = \frac{1}{K}\sum_{k=0}^{K-1}\left[\|(r_k, c_k) - (\bar{r}, \bar{c})\| - \mu_R\right]^2$$

- P_4
- P_8

$P_4 = \langle (1,0), (1,1), (2,1), (1,1), (1,2), (1,1), (0,1), (1,1) \rangle$

$\sigma_R(P_4) = 0.5$

$r \backslash c$	0	1	2
0		*	
1	*	*	*
2		*	

$(\bar{r}, \bar{c}) = (1,1) \quad \mu_R = 0.5$

$\|(1,0) - (\bar{r},\bar{c})\| - 0.5 = \|(0,-1)\| - 0.5$

$\|(1,1) - (\bar{r},\bar{c})\| - 0.5 = \|(0,0)\| - 0.5$

$\|(2,1) - (\bar{r},\bar{c})\| - 0.5 = \|(1,0)\| - 0.5$

$\|(1,1) - (\bar{r},\bar{c})\| - 0.5 = \|(0,0)\| - 0.5$

$\|(1,2) - (\bar{r},\bar{c})\| - 0.5 = \|(0,1)\| - 0.5$

$\|(1,1) - (\bar{r},\bar{c})\| - 0.5 = \|(0,0)\| - 0.5$

$\|(0,1) - (\bar{r},\bar{c})\| - 0.5 = \|(-1,0)\| - 0.5$

$\|(1,1) - (\bar{r},\bar{c})\| - 0.5 = \|(0,0)\| - 0.5$

$P_8 = \langle (1,0), (2,1), (1,2), (0,1) \rangle$

$\sigma_R(P_8) = 0$

$r \backslash c$	0	1	2
0		*	
1	*	*	*
2		*	

$(\bar{r}, \bar{c}) = (1,1) \quad \mu_R = 1$

$\|(1,0) - (\bar{r},\bar{c})\| - 1 = \|(0,-1)\| - 1$

$\|(2,1) - (\bar{r},\bar{c})\| - 1 = \|(1,0)\| - 1$

$\|(1,2) - (\bar{r},\bar{c})\| - 1 = \|(0,1)\| - 1$

$\|(0,1) - (\bar{r},\bar{c})\| - 1 = \|(-1,0)\| - 1$

DC & CV Lab.
CSIE NTU

3.2 Region Properties (cont')

- Haralick shows that μ_R/σ_R has properties:

  1. digital shape $\longrightarrow$ circular, μ_R/σ_R increases monotonically

  2. μ_R/σ_R similar for similar digital/continuous shapes

  3. orientation (rotation) and area (scale) independent

3.2 Region Properties (cont')

- Average gray level (intensity)

$$\mu = \frac{1}{A} \sum_{(r,c)\in R} I(r,c)$$

- Gray level (intensity) variance

$$\sigma^2 = \frac{1}{A} \sum_{(r,c)\in R} [I(r,c) - \mu]^2 = [\frac{1}{A} \sum_{(r,c)\in R} I(r,c)^2] - \mu^2$$

  - right hand equation lets us compute variance with only one pass

3.2 Region Properties (cont')

- microtexture properties：used to measure the texture of regions
- S: set of pixels in designated spatial relationship
- P: region's co-occurrence matrix

$$P(g_1, g_2) = \frac{\#\{[(r_1, c_1), (r_2, c_2)] \in S | I(r_1, c_1) = g_1 \text{ and } I(r_2, c_2) = g_2\}}{\#S}$$

3.2 Region Properties (cont')

- GLCM (Gray-Level Co-occurrence Matrix)

(1,1)	(1,2)	(1,3)	(1,4)
(2,1)	(2,2)	(2,3)	(2,4)
(3,1)	(3,2)	(3,3)	(3,4)
(4,1)	(4,2)	(4,3)	(4,4)

$$L_y = \{1,2,3,4\}$$
$$L_x = \{1,2,3,4\}$$

$$\begin{aligned} S = \{&((k,l),(m,n)) \in (L_y \times L_x) \times (L_y \times L_x) \mid k - m = 0, |l - n| = 1\} \\ = \{ &((1,1),(1,2)), ((1,2),(1,1)), ((1,2),(1,3)), ((1,3),(1,2)), \\ &((1,3),(1,4)), ((1,4),(1,3)), ((2,1),(2,2)), ((2,2),(2,1)), \\ &((2,2),(2,3)), ((2,3),(2,2)), ((2,3),(2,4)), ((2,4),(2,3)), \\ &((3,1),(3,2)), ((3,2),(3,1)), ((3,2),(3,3)), ((3,3),(3,2)), \\ &((3,3),(3,4)), ((3,4),(3,3)), ((4,1),(4,2)), ((4,2),(4,1)), \\ &((4,2),(4,3)), ((4,3),(4,3)), ((4,3),(4,4)), ((4,4),(4,3)) \} \end{aligned}$$

往右與往左來回都有

0 degree

Figure 9.2 The set of all distance-1 horizontal neighboring resolution cells on a 4 × 4 image.

3.2 Region Properties (cont')

0	0	1	1
0	0	1	1
0	2	2	2
2	2	3	3

		Gray Level			
		0	1	2	3
Gray Level	0	#(0,0)	#(0,1)	#(0,2)	#(0,3)
	1	#(1,0)	#(1,1)	#(1,2)	#(1,3)
	2	#(2,0)	#(2,1)	#(2,2)	#(2,3)
	3	#(3,0)	#(3,1)	#(3,2)	#(3,3)

$$0^\circ \quad P_H = \begin{pmatrix} 4 & 2 & 1 & 0 \\ 2 & 4 & 0 & 0 \\ 1 & 0 & 6 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix} \qquad 90^\circ \quad P_V = \begin{pmatrix} 6 & 0 & 2 & 0 \\ 0 & 4 & 2 & 0 \\ 2 & 2 & 2 & 2 \\ 0 & 0 & 2 & 0 \end{pmatrix}$$

$$135^\circ \quad P_{LD} = \begin{pmatrix} 2 & 1 & 3 & 0 \\ 1 & 2 & 1 & 0 \\ 3 & 1 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{pmatrix} \qquad 45^\circ \quad P_{SD} = \begin{pmatrix} 4 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 \\ 0 & 2 & 4 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Figure 9.3 Spatial co-occurrence calculations (Haralick, Shanmugam, and Dinstein, 1973).

$$0^\circ \quad P_H = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 4 & \boxed{2} & 1 & 0 \\ \boxed{2} & 4 & 0 & 0 \\ 1 & 0 & \boxed{6} & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix} \end{array}$$

$P(1,0)$

$P(0,1)$

$P(2,2)$

0	0	1	1
0	0	1	1
0	2	2	2
2	2	3	3

3.2 Region Properties (cont')

- Texture properties example：

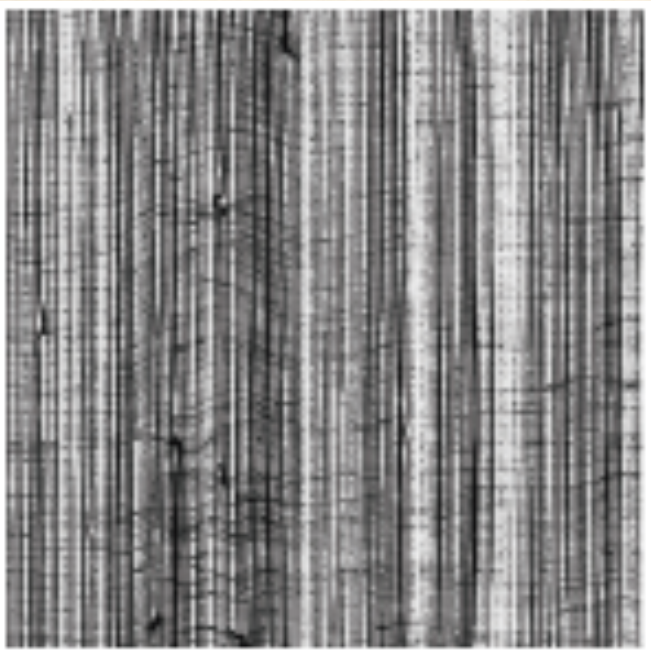
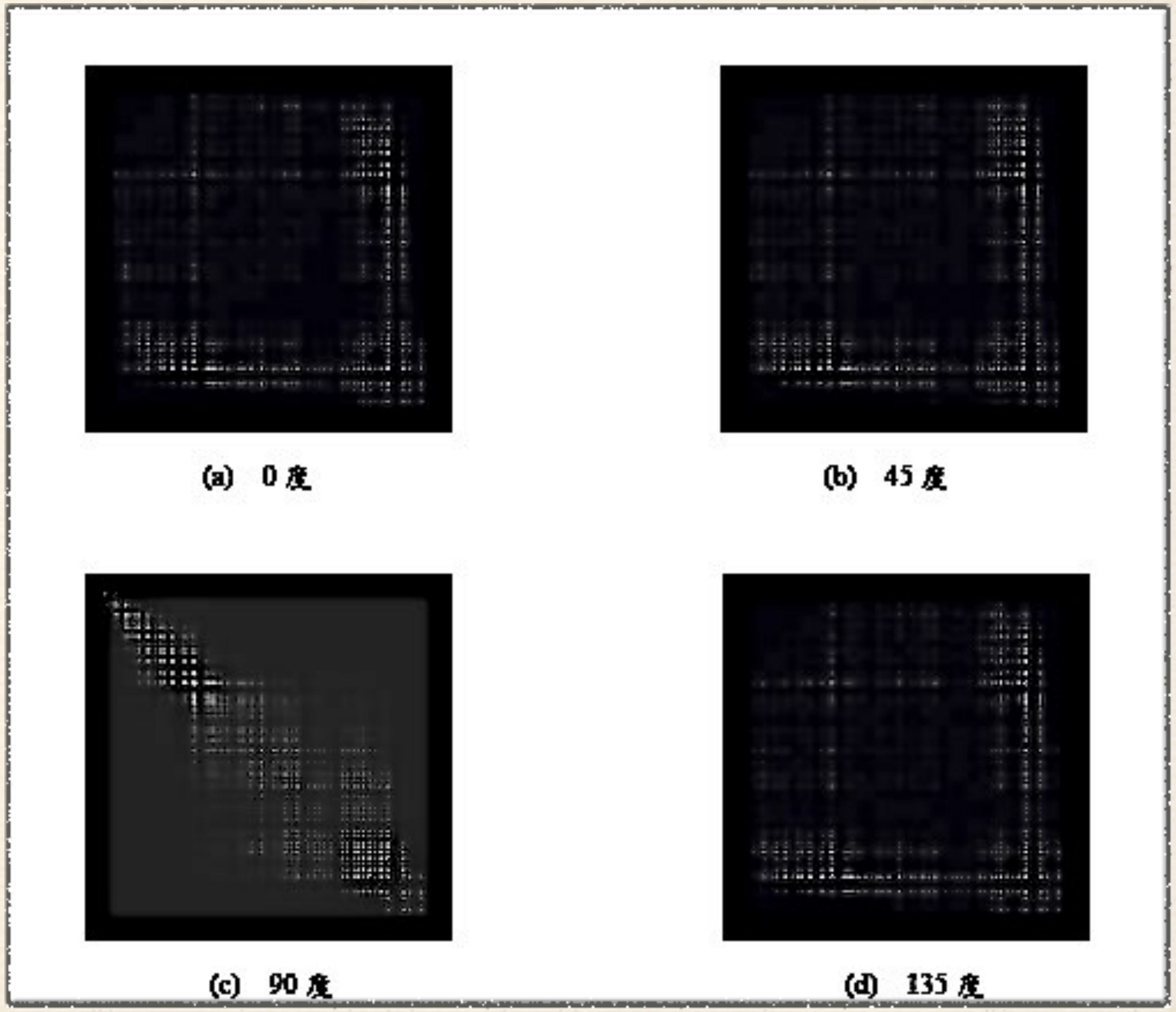
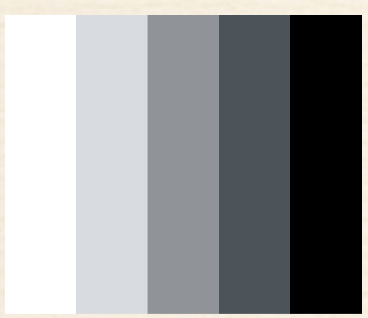


圖 2.4 結構性紋理影像實例



3.2 Region Properties (cont')

- Texture properties extreme example：



Gray level value
(approximation)

4	3	2	1	0
4	3	2	1	0
4	3	2	1	0
4	3	2	1	0
4	3	2	1	0

90 degree

8	0	0	0	0
0	8	0	0	0
0	0	8	0	0
0	0	0	8	0
0	0	0	0	8

3.2 Region Properties (cont')

- texture second moment (Haralick, Shanmugam, and Dinstein, 1973)

$$M = \sum_{g_1, g_2} P^2(g_1, g_2)$$

圖 2.8 高度一致性與均勻性影像實例

圖 2.9 高度一致性與均勻性影像之 GLCM

- texture entropy

$$E = -\sum_{g_1, g_2} P(g_1, g_2) \log P(g_1, g_2)$$

圖 2.12 高複雜度影像實例

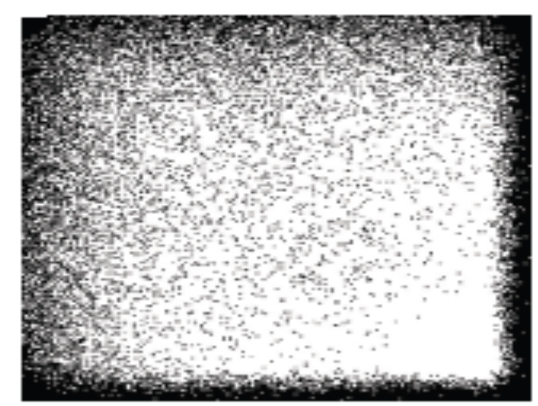


圖 2.13 高複雜度影像之 GLCM

3.2 Region Properties (cont')

- texture contrast

$$C = \sum_{g_1} \sum_{g_2} |g_1 - g_2| P(g_1, g_2)$$

圖 2.6 高對比度影像實例

圖 2.7 高對比度影像之 GLCM

- texture homogeneity

$$H = \sum_{g_1} \sum_{g_2} \frac{P(g_1, g_2)}{k + |g_1 - g_2|}$$

where *k* is some small constant

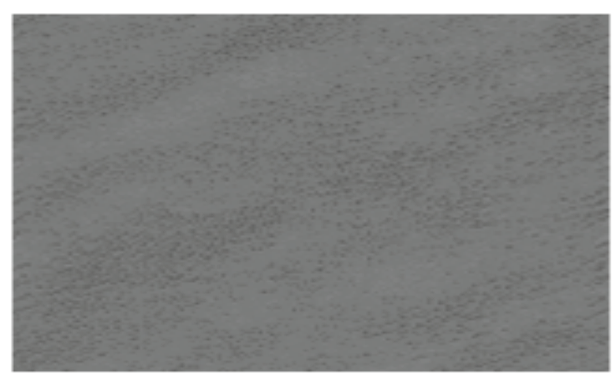


圖 2.10 高同質性影像實例

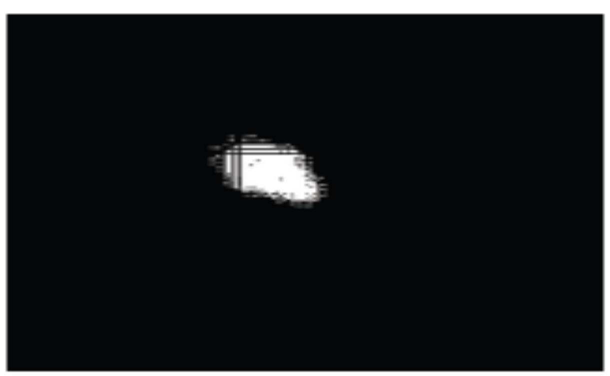


圖 2.11 高同質性影像之 GLCM

3.2.1 Extremal Points

- Extremal Points
  - Definition
    1. points
    2. axes
    3. length
    4. orientation
  - Three cases：linelike shape

    ① Triangular shape

    ② square and rectangular shape

    ③ octagonal shape

3.2.1 Extremal Points (cont')

- eight distinct extremal pixels:
  1. topmost right
  2. rightmost top
  3. rightmost bottom
  4. bottommost right
  5. bottommost left
  6. leftmost bottom
  7. leftmost top
  8. topmost left

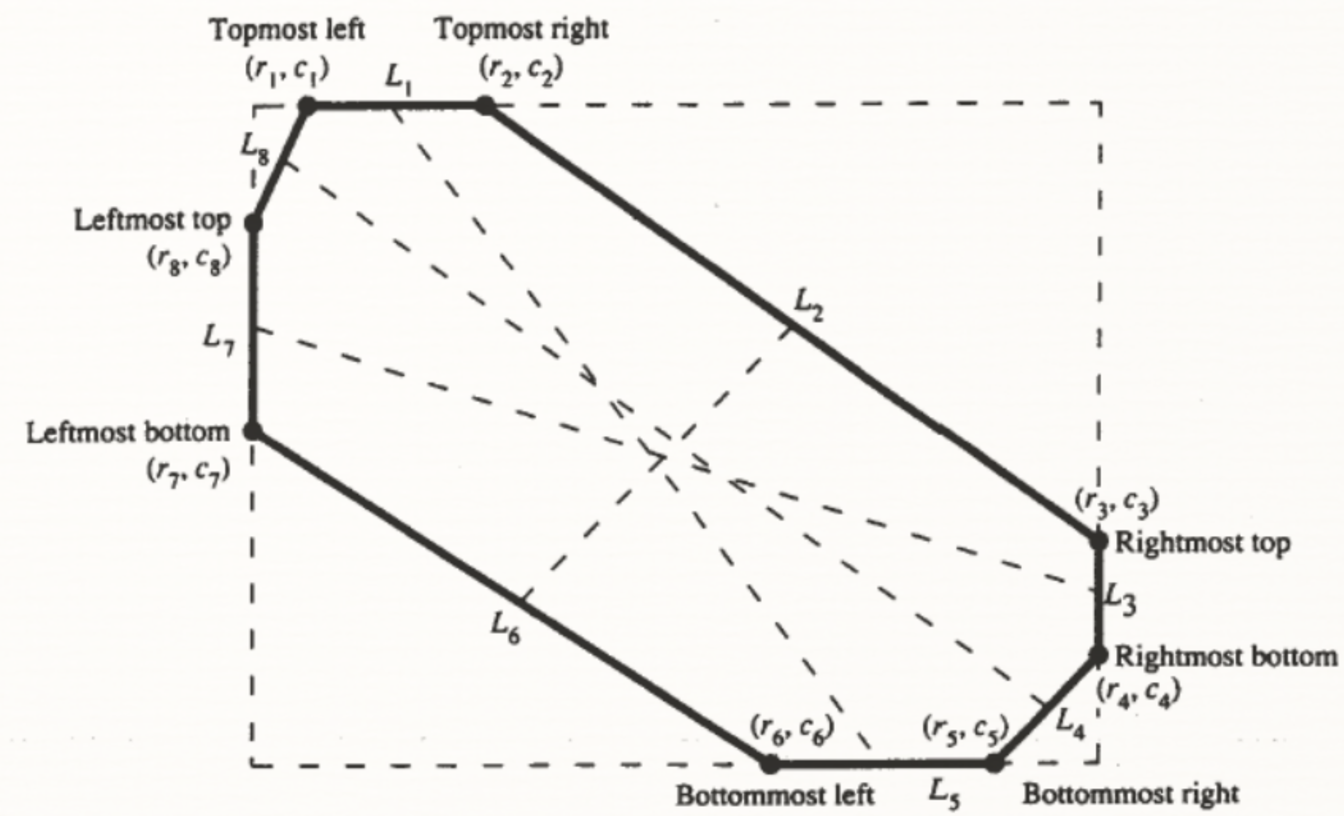


Figure 3.1 The eight extremal points a region can have and the normally oriented bounding rectangle that encloses the region. The interior dotted lines pair together opposite sides.

3.2.1 Extremal Points (cont')

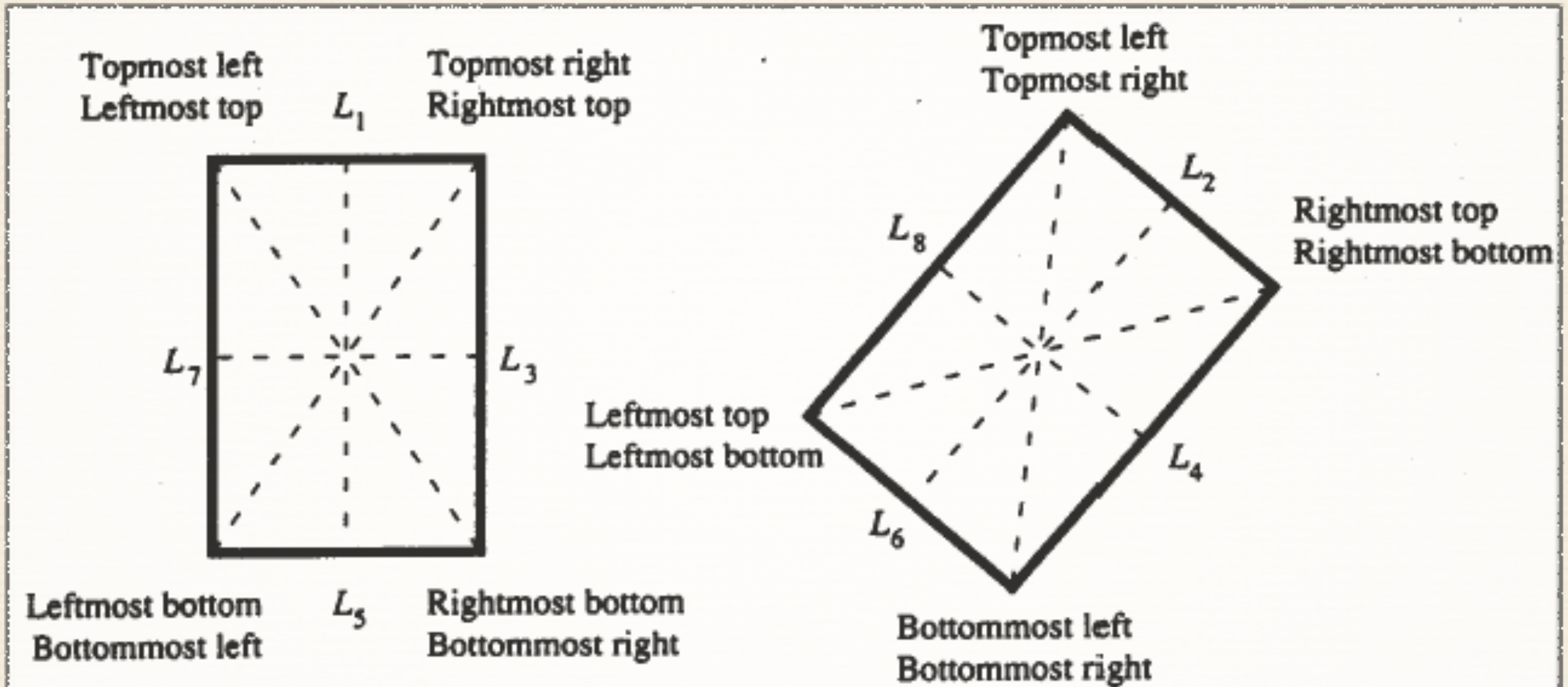


Figure 3.2 Two regions in which the extremal points are not unique and in which they pair differently. The interior dotted lines pair together opposite sides. Because some extremal points are coincident, some opposite sides have zero length.

3.2.1 Extremal Points (cont')

- association of the name of the eight extremal points with their coordinates

Table 3.1 Association of the name of the eight extremal points with their coordinate representation.

Name of Extremal Point	Coordinate Representation
Topmost left	(r_1, c_1)
Topmost right	(r_2, c_2)
Rightmost top	(r_3, c_3)
Rightmost bottom	(r_4, c_4)
Bottommost right	(r_5, c_5)
Bottommost left	(r_6, c_6)
Leftmost bottom	(r_7, c_7)
Leftmost top	(r_8, c_8)

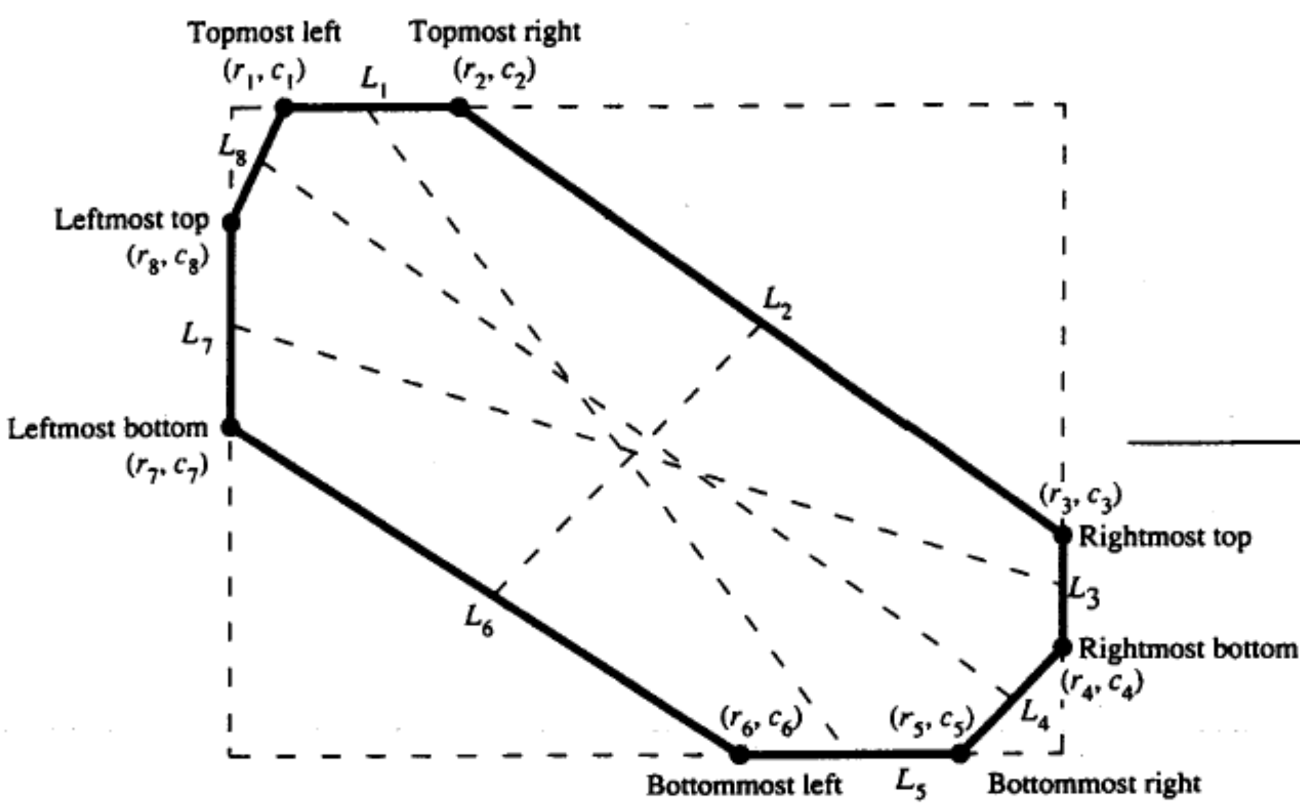


Figure 3.1 The eight extremal points a region can have and the normally oriented bounding rectangle that encloses the region. The interior dotted lines pair together opposite sides.

3.2.1 Extremal Points (cont')

- association of the name of an external coordinate with its definition：

Table 3.2 Association of the name of an extremal coordinate with its definition.

Name of Extremal Coordinate	Coordinate Representation and Definition
Topmost row	$\text{rmin} = \min\{r \mid (r, c) \in R\}$
Bottommost row	$\text{rmax} = \max\{r \mid (r, c) \in R\}$
Leftmost column	$\text{cmin} = \min\{c \mid (r, c) \in R\}$
Rightmost column	$\text{cmax} = \max\{c \mid (r, c) \in R\}$

- directly define the coordinates of the extremal points:

$$r_1 = r_2 = rmin \qquad r_5 = r_6 = rmax$$

$$c_1 = \min\{c \mid (rmin, c) \in R\} \quad c_5 = \max\{c \mid (rmax, c) \in R\}$$

$$c_2 = \max\{c \mid (rmin, c) \in R\} \quad c_6 = \min\{c \mid (rmax, c) \in R\}$$

$$r_3 = \min\{r \mid (r, cmax) \in R\} \quad r_7 = \max\{r \mid (r, cmin) \in R\}$$

$$r_4 = \max\{r \mid (r, cmax) \in R\} \quad r_8 = \min\{r \mid (r, cmin) \in R\}$$

$$c_3 = c_4 = cmax \qquad c_7 = c_8 = cmin$$

(r,c)=(0,0)
start from here

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	1	0
2	0	0	1	1	1	1	1	0
3	0	0	1	1	1	0	0	0
4	0	0	1	1	1	0	0	0
5	0	1	1	1	1	1	0	0
6	0	1	1	0	0	1	1	0
7	0	0	0	0	0	0	0	0

3.2.1 Extremal Points (cont')

- extremal points occur in opposite pairs:
  - topmost left ⟷ bottommost right
  - topmost right ⟷ bottommost left
  - rightmost top ⟷ leftmost bottom
  - rightmost bottom ⟷ leftmost top
- each opposite extremal point pair: defines an axis
- axis properties: length, orientation

3.2.1 Extremal Points (cont')

- orientation taken counterclockwise w.r.t. column (horizontal) axis

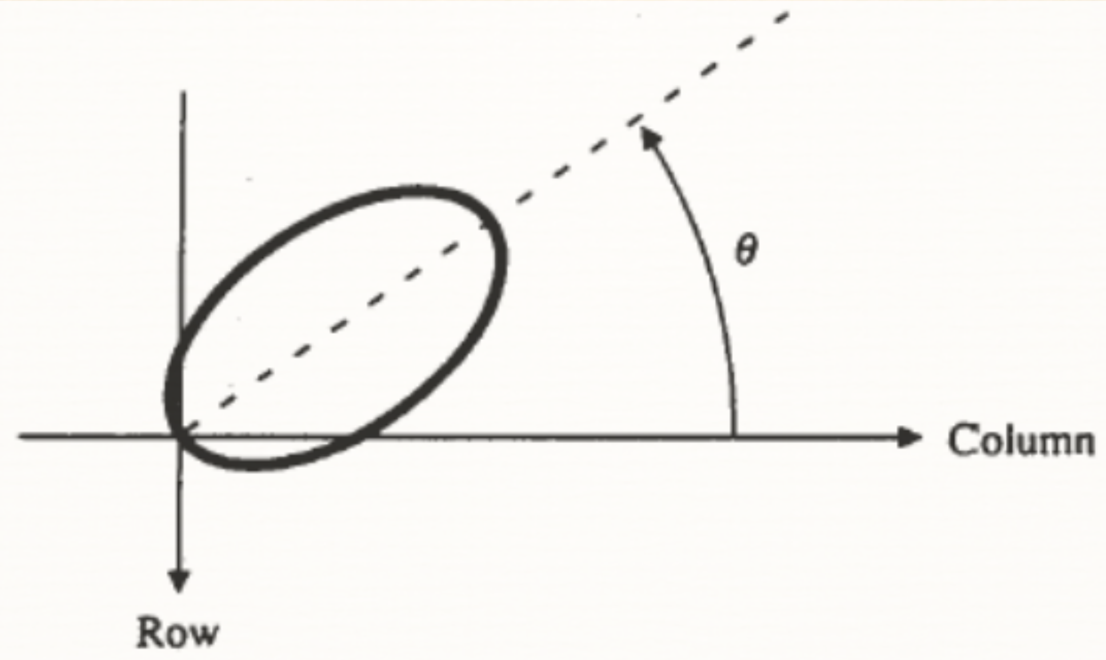
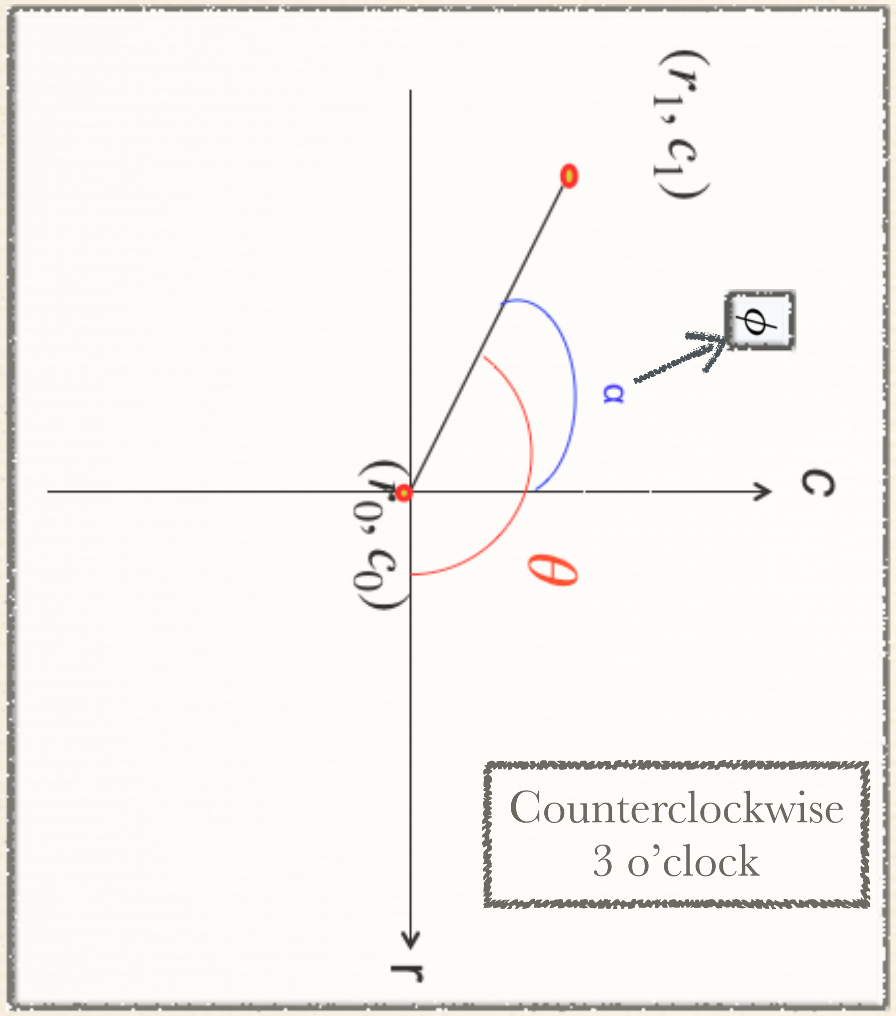


Figure 3.3 The eight extremal points a region can have and the normally oriented bounding rectangle that encloses the region. The interior dotted lines pair together opposite extremal points. They constitute the axes M_1, M_2, M_3, and M_4.

3.2.1 Extremal Points (cont')



(r,c)≠(x,y)
(r,c)=(down to positive, right to positive)
r0>r1，c1>c0

- Goal: α = ?

$$\tan\theta = \frac{c_0 - c_1}{r_0 - r_1}$$

$$\tan\theta = \tan(90 + \alpha) = -\cot\alpha$$

$$\cot\alpha = -\frac{c_0 - c_1}{r_0 - r_1}$$

$$\tan\alpha = -\frac{r_0 - r_1}{c_0 - c_1}$$

$$\alpha = \tan^{-1}\left(-\frac{r_0 - r_1}{c_0 - c_1}\right)$$

3.2.1 Extremal Points (cont')

- orientation convention for the axes
- axes paired : M1 with M3 and M2 with M4

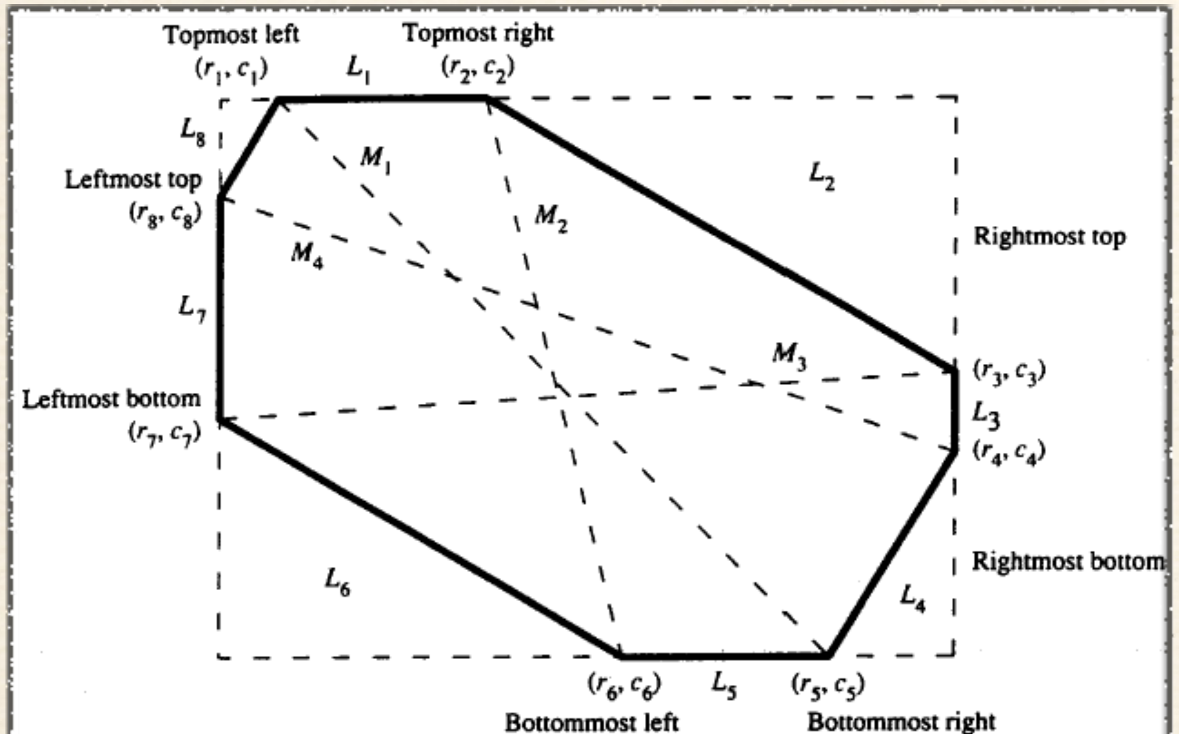


Figure 3.4 Orientation convention for the axes. The orientation angle of an axis is measured counterclockwise from the column axis.

$$\phi_1 = \tan^{-1} \frac{r_1 - r_5}{-(c_1 - c_5)}$$

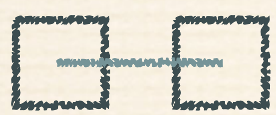
$$\phi_2 = \tan^{-1} \frac{r_2 - r_6}{-(c_2 - c_6)}$$

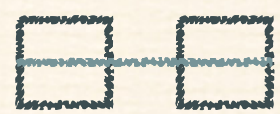
$$\phi_3 = \tan^{-1} \frac{r_3 - r_7}{-(c_3 - c_7)}$$

$$\phi_4 = \tan^{-1} \frac{r_4 - r_8}{-(c_4 - c_8)}$$

3.2.1 Extremal Points (cont')

- the length covered by two pixels horizontally adjacent
  1. distance between pixel centers
  2. from left edge of left pixel to right edge of right pixel

1. 

2. 

- distance calculation: add a small increment to the Euclidean distance

$$M_1 = \sqrt{(r_1 - r_5)^2 + (c_1 - c_5)^2} + Q(\phi_1)$$

$$M_2 = \sqrt{(r_2 - r_6)^2 + (c_2 - c_6)^2} + Q(\phi_2)$$

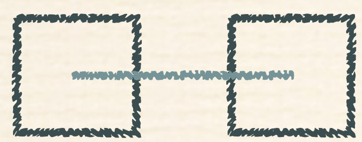
$$M_3 = \sqrt{(r_3 - r_7)^2 + (c_3 - c_7)^2} + Q(\phi_3)$$

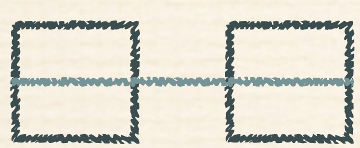
$$M_4 = \sqrt{(r_4 - r_8)^2 + (c_4 - c_8)^2} + Q(\phi_4)$$

M_1: axis between (r_1, c_1) and (r_5, c_5)
M_2: axis between (r_2, c_2) and (r_6, c_6)
M_3: axis between (r_3, c_3) and (r_7, c_7)
M_4: axis between (r_4, c_4) and (r_8, c_8)

3.2.1 Extremal Points (cont')

- length going from left edge of left pixel to right edge of right pixel

  - 1. 

  - 2. 

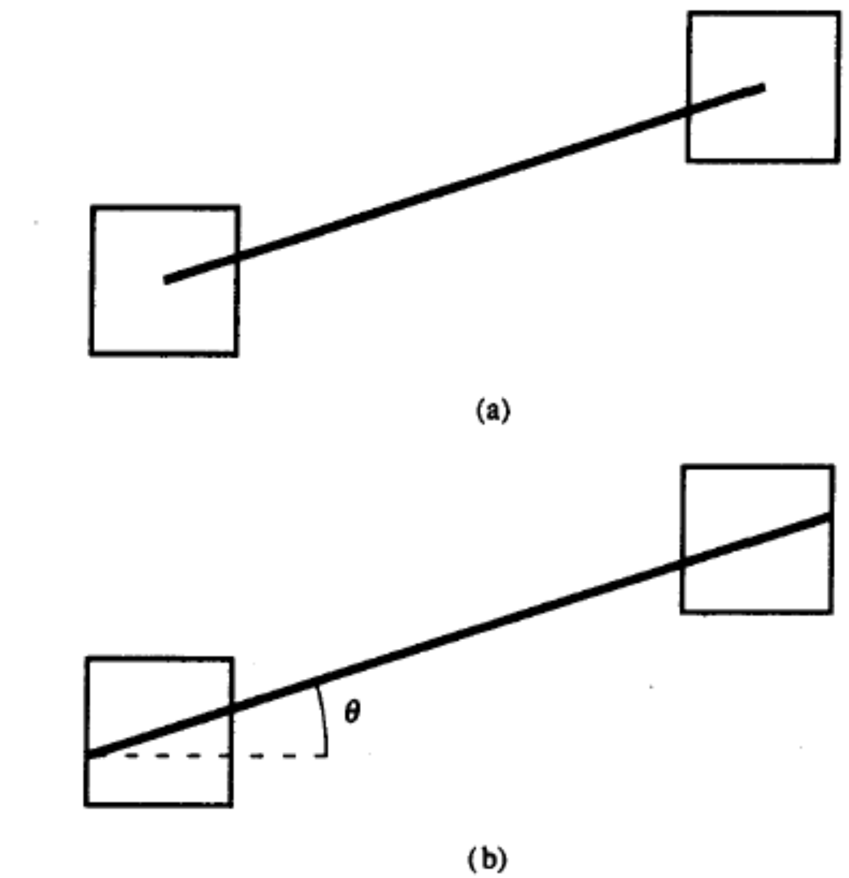


Figure 3.5 Diagram showing why the distance between two pixels must be increased if it is to count length going from the left edge of the left pixel to the right edge of the right pixel. Part (a) shows the distance between pixel centers; (b) shows the left edge to right edge distance. Each pixel must add a length that is the length of the hypotenuse of a right triangle having base 1/2. For $|\theta| < 45°$, this length is $1/2 \cos\theta$ for each pixel.

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3.2.1 Extremal Points (cont')

- length going from left edge of left pixel to right edge of right pixel (cont')

$$Q(\theta) = \begin{cases} \frac{1}{|\cos\theta|} & \text{if } |\theta| < 45^\circ \\ \frac{1}{|\sin\theta|} & \text{if } |\theta| > 45^\circ \end{cases}$$

$$M_1 = \sqrt{(r_1 - r_5)^2 + (c_1 - c_5)^2} + Q(\phi_1)$$

$$M_2 = \sqrt{(r_2 - r_6)^2 + (c_2 - c_6)^2} + Q(\phi_2)$$

$$M_3 = \sqrt{(r_3 - r_7)^2 + (c_3 - c_7)^2} + Q(\phi_3)$$

$$M_4 = \sqrt{(r_4 - r_8)^2 + (c_4 - c_8)^2} + Q(\phi_4)$$

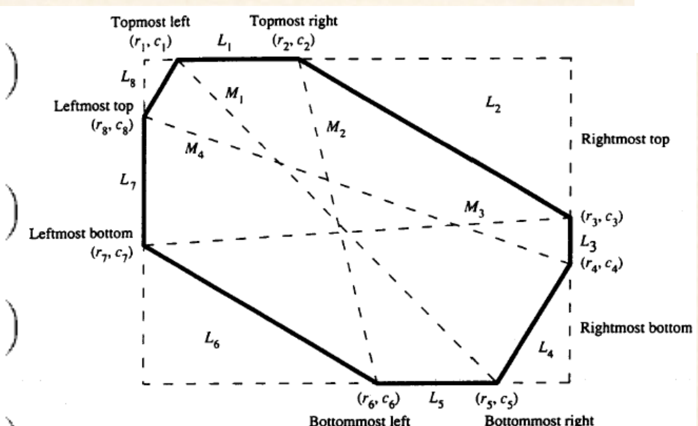
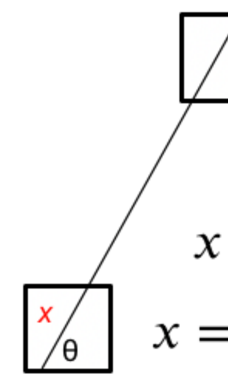


Figure 3.4 Orientation convention for the axes. The orientation angle of an axis is measured counterclockwise from the column axis.



$$x * \sin\theta = 1$$

$$x = \frac{1}{\sin\theta}$$

- distance between i th and j th extremal point

$$M_{ij} = \sqrt{(r_i - r_j)^2 + (c_i - c_j)^2} + 1.12$$

- average value of Q (θ) = 1.12, largest error 0.294 = $\sqrt{2}$ -1.12

3.2.1 Extremal Points (cont')

- How can we use extremal points?

    1. Line's length/orientation
    2. Triangle's base/height
    3. Rectangle's orientation

3.2.1 Extremal Points (cont')

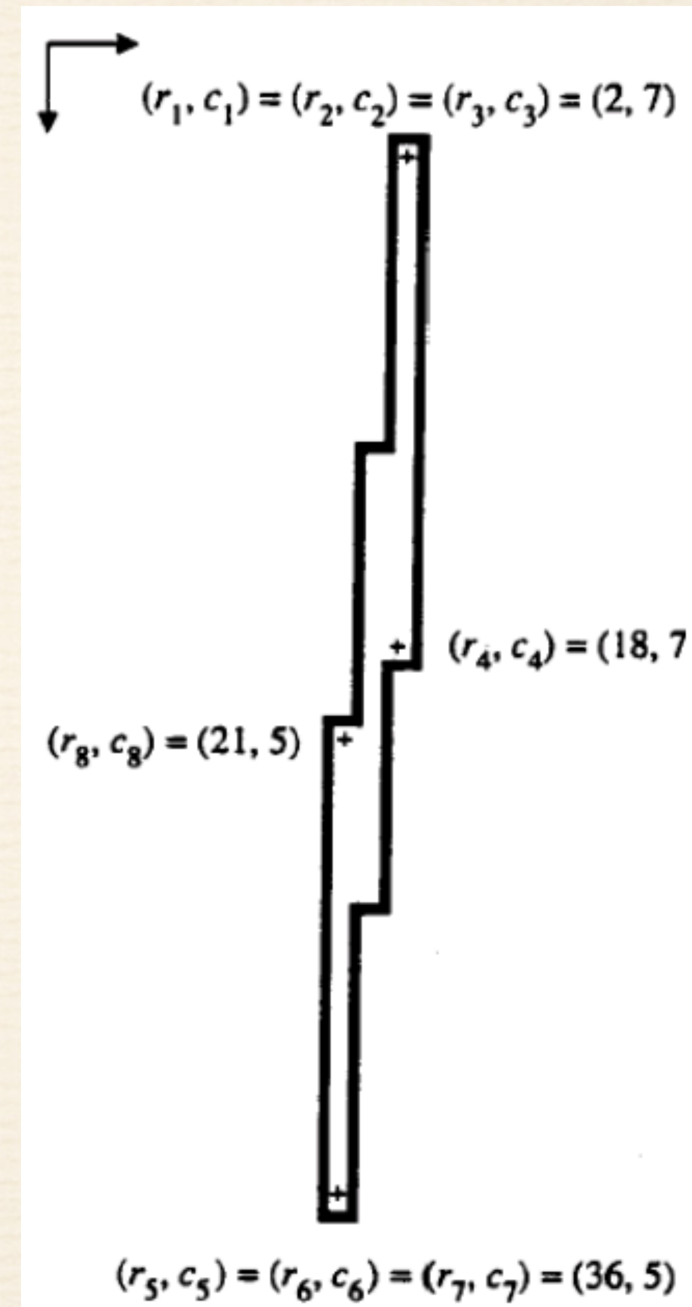
- calculation of the axis length and orientation of a linelike shape

$$M_1 = M_2 = M_3 = \sqrt{(2-36)^2 + (7-5)^2)} + 1.12 = 35.18$$

$$M_4 = \sqrt{(18-21)^2 + (7-5)^2} + 1.12 = 4.73$$

$$\phi_1 = \phi_2 = \phi_3 = \tan^{-1} \frac{-34}{-(2)} = -93.37°$$

Figure 3.6 Calculation of the axis length and orientation of a linelike shape.



3.2.1 Extremal Points (cont')

- calculations for length of sides base and altitude for a triangle

$$M_{13} = M_{23} = \sqrt{(6-26)^2 + (7-9)^2} + 1.12 = 21.22$$

$$M_{14} = M_{24} = \sqrt{(6-35)^2 + (7-9)^2} + 1.12 = 30.19$$

$$M_{15} = M_{25} = M_{16} = M_{26} = M_{17} = M_{27} = \sqrt{(6-36)^2 + (7-6)^2} + 1.12 = 31.14$$

$$M_{18} = M_{28} = \sqrt{(6-21)^2 + (7-6)^2} + 1.12 = 16.15$$

$$M_{34} = \sqrt{(26-35)^2 + (9-9)^2} + 1.12 = 10.12$$

$$M_{35} = M_{36} = M_{37} = \sqrt{(26-36)^2 + (9-6)^2} + 1.12 = 11.56$$

$$M_{38} = \sqrt{(26-21)^2 + (9-6)^2} + 1.12 = 6.95$$

$$M_{45} = M_{46} = M_{47} = \sqrt{(35-36)^2 + (9-6)^2} + 1.12 = 4.28$$

$$M_{48} = \sqrt{(35-21)^2 + (9-6)^2} + 1.12 = 15.44$$

$$M_{58} = M_{68} = M_{78} = \sqrt{(36-21)^2 + (6-6)^2} + 1.12 = 16.42$$

$k_1 = 1, \quad k_2 = 4, \quad k_3 = 5$ Let k_1, k_2, and k_3 be any indices maximizing $M_{k_1k_2} + M_{k_1k_3}$

$L = 30.67 = (M_{14} + M_{15})/2$

$B = 4.28 = M_{45}$

$h = 30.60 \qquad h = \sqrt{L^2 - \left(\frac{B}{2}\right)^2}$

$$\phi_h = \tan^{-1} \frac{\frac{1}{2}(35+36)-6}{-(\frac{1}{2}(9+6)-7)} \qquad \phi_h = \tan^{-1} \frac{\frac{1}{2}(r_{k_2}+r_{k_3}) - r_{k_1}}{-[\frac{1}{2}(c_{k_2}+c_{k_3}) - c_{k_1}]}$$

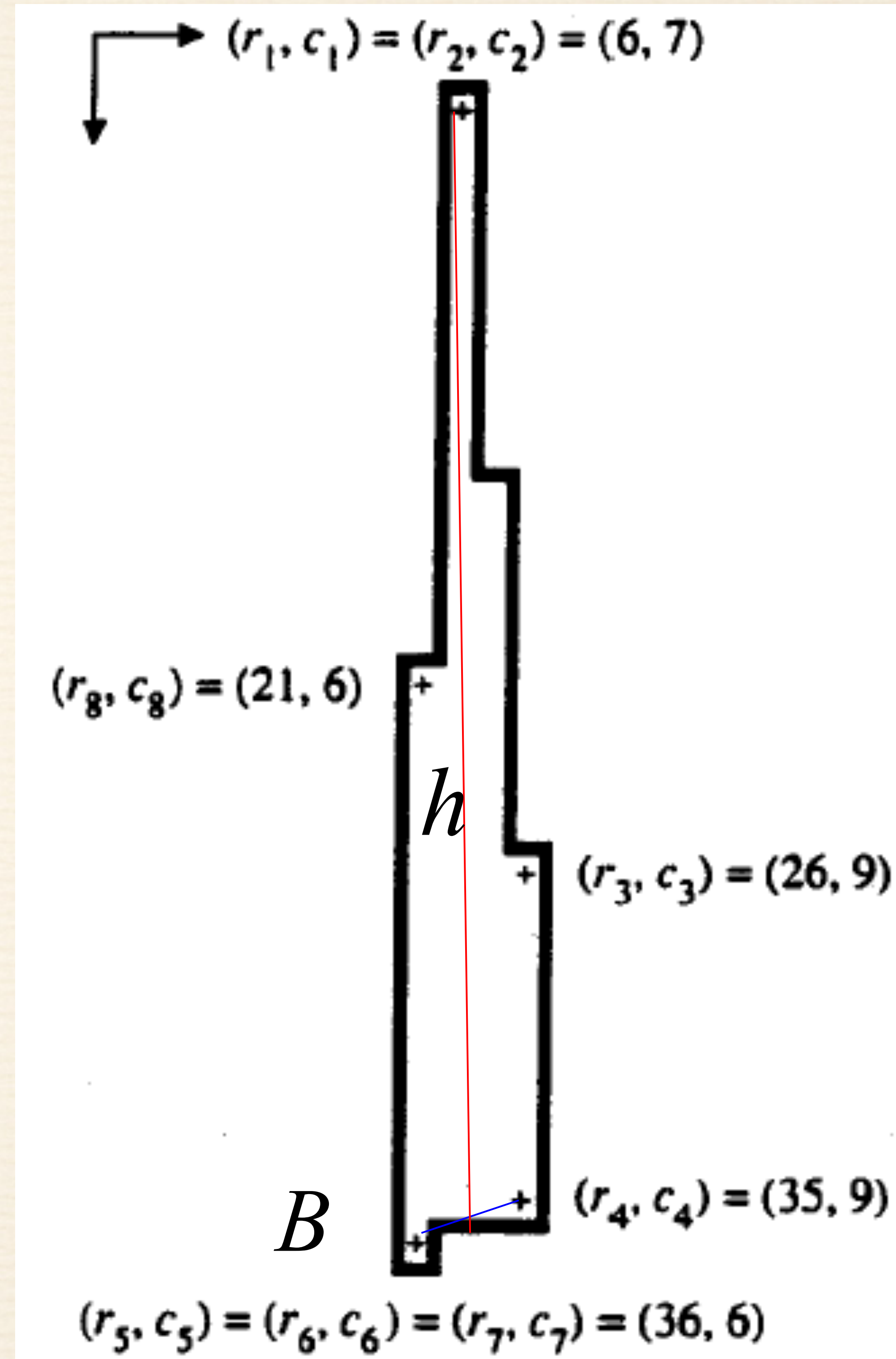


Figure 3.7 Calculations for length of sides, base, and altitude for an example isosceles triangle.

3.2.1 Extremal Points (cont')

- geometry of the tilted rectangle Line's length/orientation

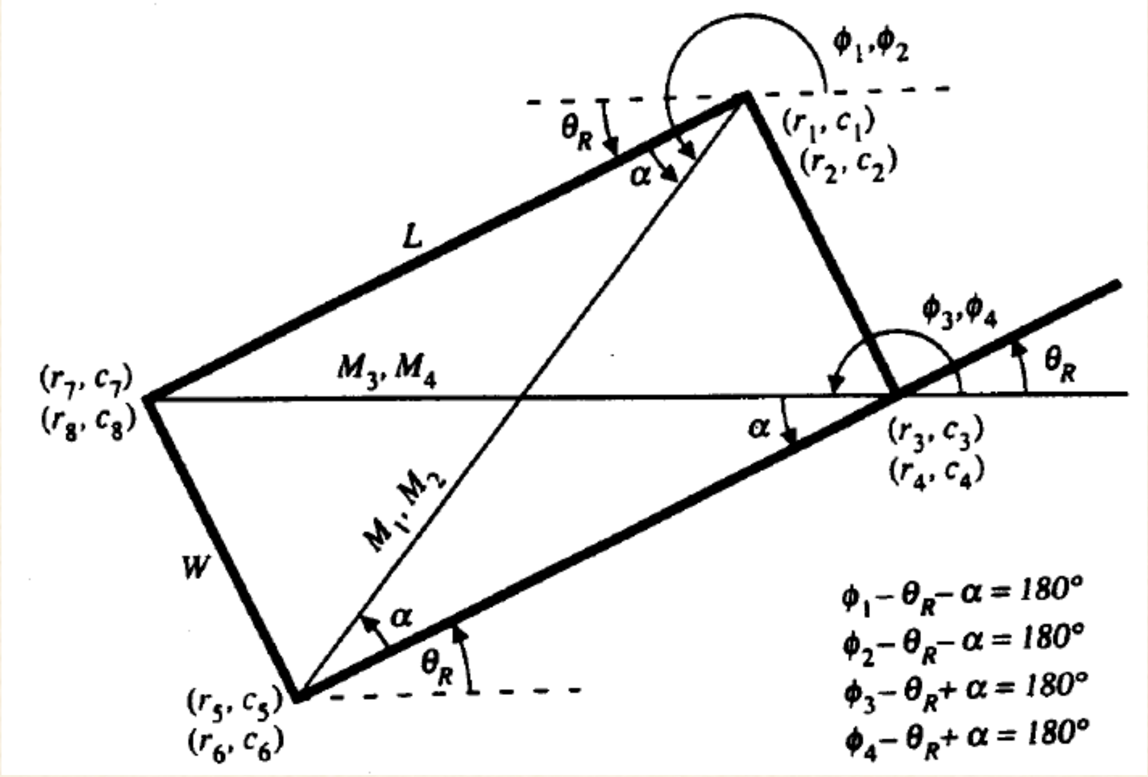


Figure 3.8 Geometry of the tilted rectangle. The diagram shows the relationship between the angular orientation θ_R and the angles of the axes joining opposite pairs of extremal points. Here (r_1, c_1) is the topmost vertex; (r_3, c_3) the rightmost vertex; (r_5, c_5) the bottommost vertex; and (r_7, c_7) the leftmost vertex.

3.2.1 Extremal Points (cont')

- calculation for the orientation of an example rectangle

$$M_1 = \sqrt{(9-37)^2 + (9-38)^2} + 1.12 = 41.43$$

$$M_2 = \sqrt{(9-37)^2 + (16-31)^2} + 1.12 = 32.88$$

$$M_3 = \sqrt{(13-33)^2 + (41-6)^2} + 1.12 = 41.43$$

$$M_4 = \sqrt{(20-33)^2 + (41-6)^2} + 1.12 = 38.46$$

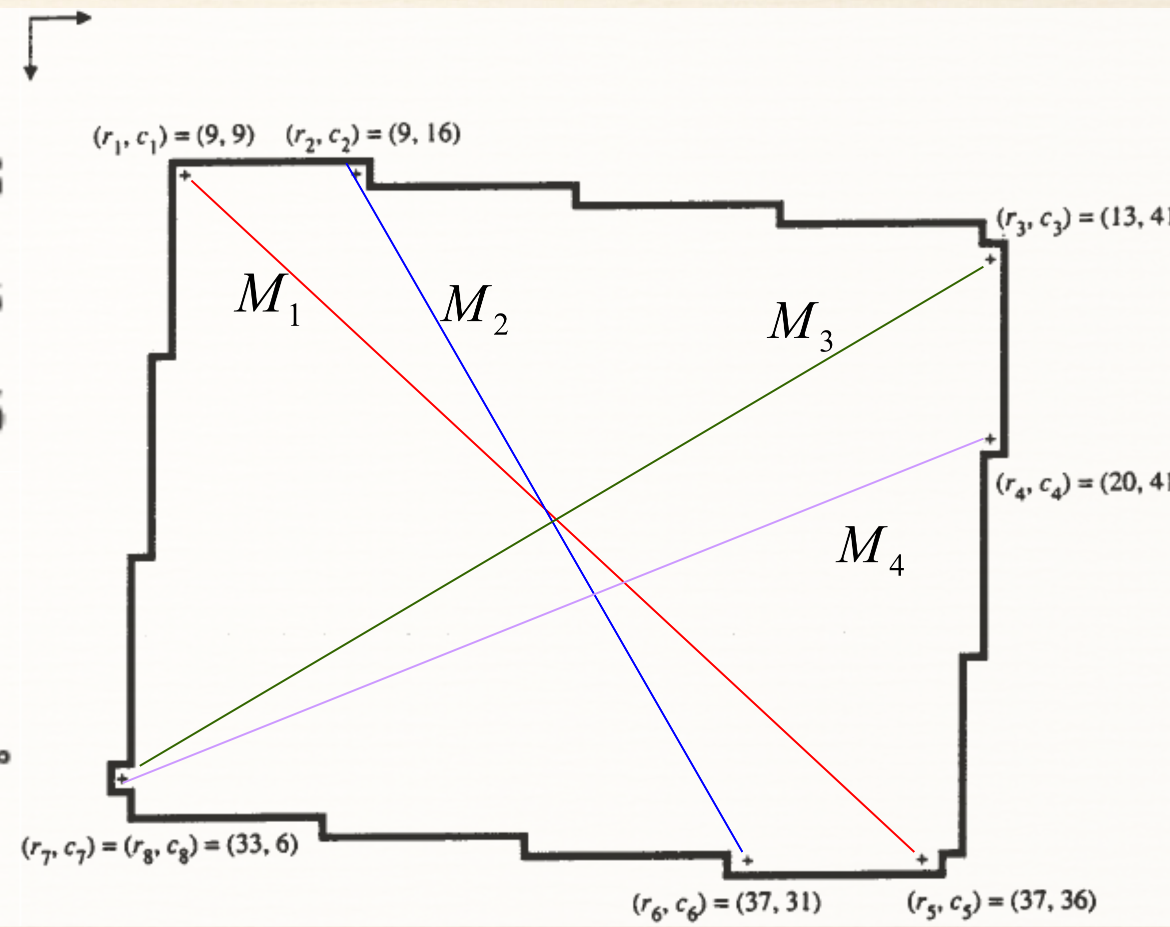
$$M_{(1)} = M_1$$

$$M_{m(1)} - M_3$$

$$\phi_{(1)} = \tan^{-1} \frac{r_1 - r_5}{-(c_1 - c_5)} = \tan^{-1} \frac{9-37}{-(9-38)} \quad \tan^{-1} \frac{-28}{29} = -43.99^\circ$$

$$\phi_{m(1)} = \tan^{-1} \frac{r_3 - r_7}{-(c_3 - c_7)} = \tan^{-1} \frac{13-33}{-(41-6)} \quad \tan^{-1} \frac{-20}{-35} = -150.26^\circ$$

$$\theta_R = \frac{\phi_{(1)} + \phi_{m(1)}}{2} - 180^\circ = 82.86^\circ$$



$$\boxed{\theta_R = \frac{\phi_1 + \phi_{m1}}{2} - 180}$$

$$\boxed{\tan\theta = \tan(\theta + 180^\circ)}$$

3.2.1 Extremal Points (cont')

- axes and their mates that arise from octagonal-shaped regions

Side length	Extremal point
$L_1 = 3$	$(r_1, c_1) = (2, 13)$
$L_2 = 5.51$	$(r_2, c_2) = (2, 15)$
$L_3 = 3$	$(r_3, c_3) = (6, 17)$
$L_4 = 10.55$	$(r_4, c_4) = (8, 17)$
$L_5 = 5$	$(r_5, c_5) = (13, 9)$
$L_6 = 4.28$	$(r_6, c_6) = (13, 5)$
$L_7 = 6$	$(r_7, c_7) = (10, 4)$
$L_8 = 10.6$	$(r_8, c_8) = (5, 4)$

Axis length	Axis orientation
$A_1: 5.295$	0°
$A_2: 4.935$	−66.9°
$A_3: 4.5$	90°
$A_4: 10.575$	25.2°

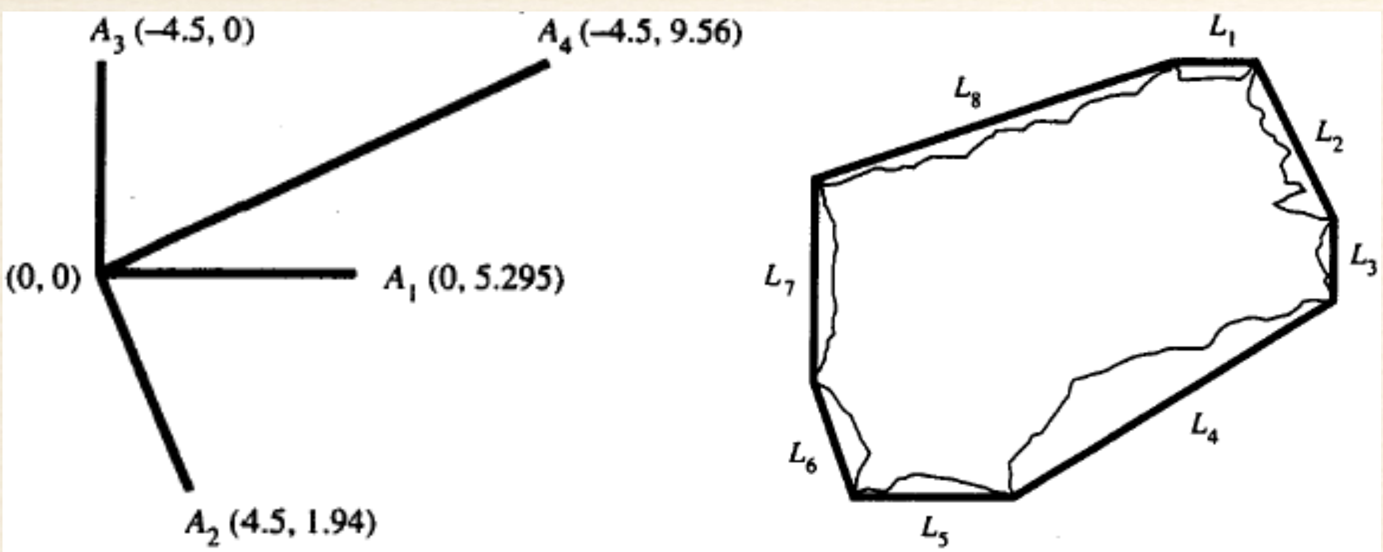


Figure 3.10 Axes and their mates that arise from octagonal-shaped regions and their extremal points.

$$L_1 = |c_1 - c_2| + 1$$
$$L_2 = \sqrt{(r_2 - r_3)^2 + (c_2 - c_3)^2} + Q(\theta_2)$$
$$L_3 = |r_3 - r_4| + 1$$
$$L_4 = \sqrt{(r_4 - r_5)^2 + (c_4 - c_5)^2} + Q(\theta_4)$$
$$L_5 = |c_5 - c_6| + 1$$
$$L_6 = \sqrt{(r_6 - r_7)^2 + (c_6 - c_7)^2} + Q(\theta_2)$$
$$L_7 = |r_7 - r_8| + 1$$
$$L_8 = \sqrt{(r_8 - r_1)^2 + (c_8 - c_1)^2} + Q(\theta_4)$$

$$A_1 = \frac{(L_1 + L_5)}{2}$$
$$A_2 = \frac{(L_2 + L_6)}{2}$$
$$A_3 = \frac{(L_3 + L_7)}{2}$$
$$A_4 = \frac{(L_4 + L_8)}{2}$$

3.2.2 Spatial Moments

- Second-order row moment

$$\mu_{rr} = \frac{1}{A} \sum_{(r,c)\in R} (r - \bar{r})^2$$

- Second-order mixed moment

$$\mu_{rc} = \frac{1}{A} \sum_{(r,c)\in R} (r - \bar{r})(c - \bar{c})$$

- Second-order column moment

$$\mu_{cc} = \frac{1}{A} \sum_{(r,c)\in R} (c - \bar{c})^2$$

3.2.3 Mixed Spatial Gray Level Moments

- region properties: position, extent, shape, gray level properties

- Second-order mixed gray level spatial moments

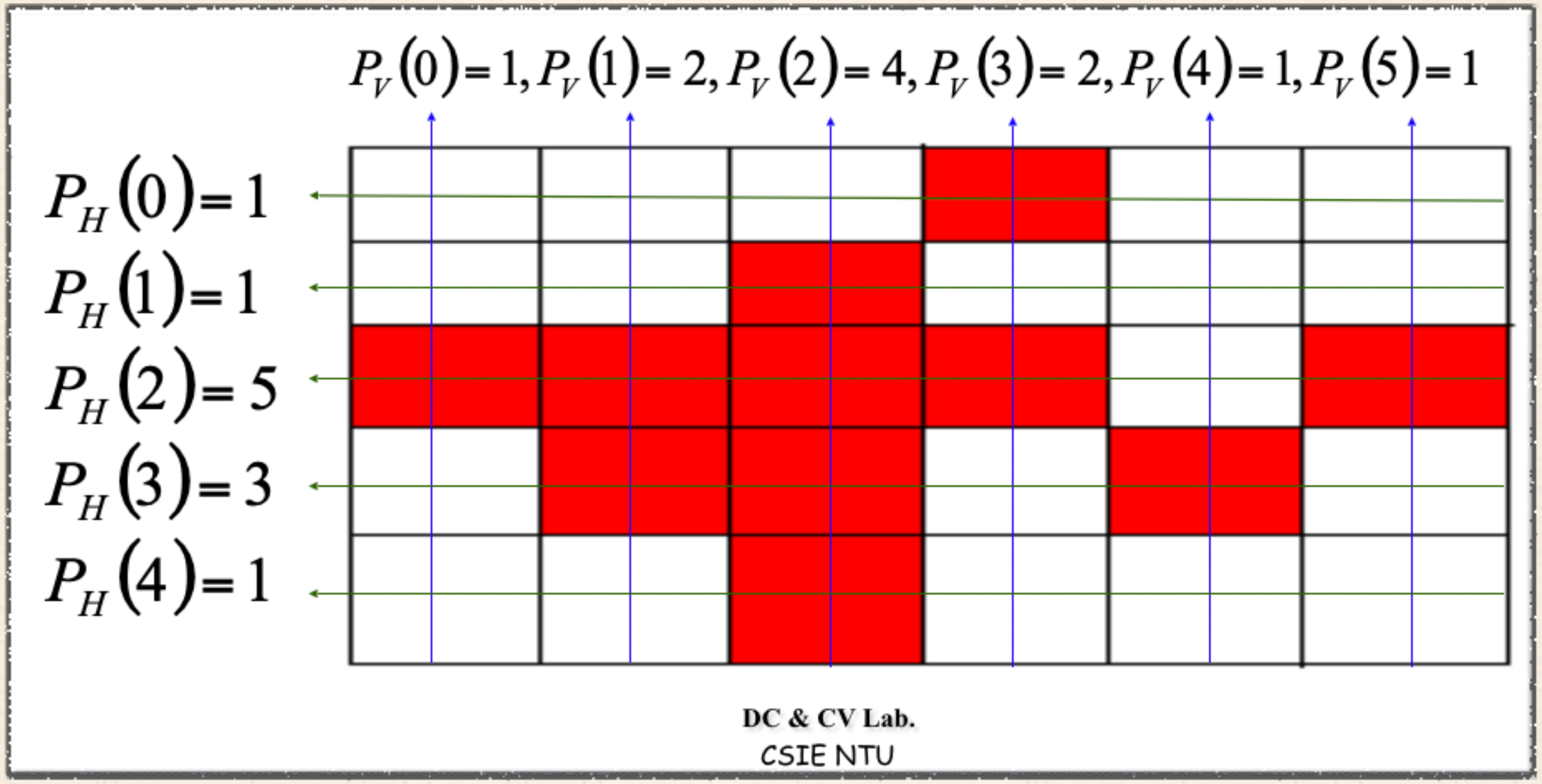
$$\mu_{rg} = \frac{1}{A} \sum_{(r,c)\in R} (r - \bar{r})[I(r,c) - \mu]$$

$$\mu_{cg} = \frac{1}{A} \sum_{(r,c)\in R} (c - \bar{c})[I(r,c) - \mu]$$

3.3 Signature Properties

3.3 Signature Properties (cont')

- vertical projection $P_V(c) = \#\{r|(r,c) \in R\}$
- horizontal projection $P_H(r) = \#\{c|(r,c) \in R\}$



3.3 Signature Properties (cont')

- diagonal projection from lower left to upper right

$$P_D(d) = \#\{(r,c) \in R | r + c = d\}$$

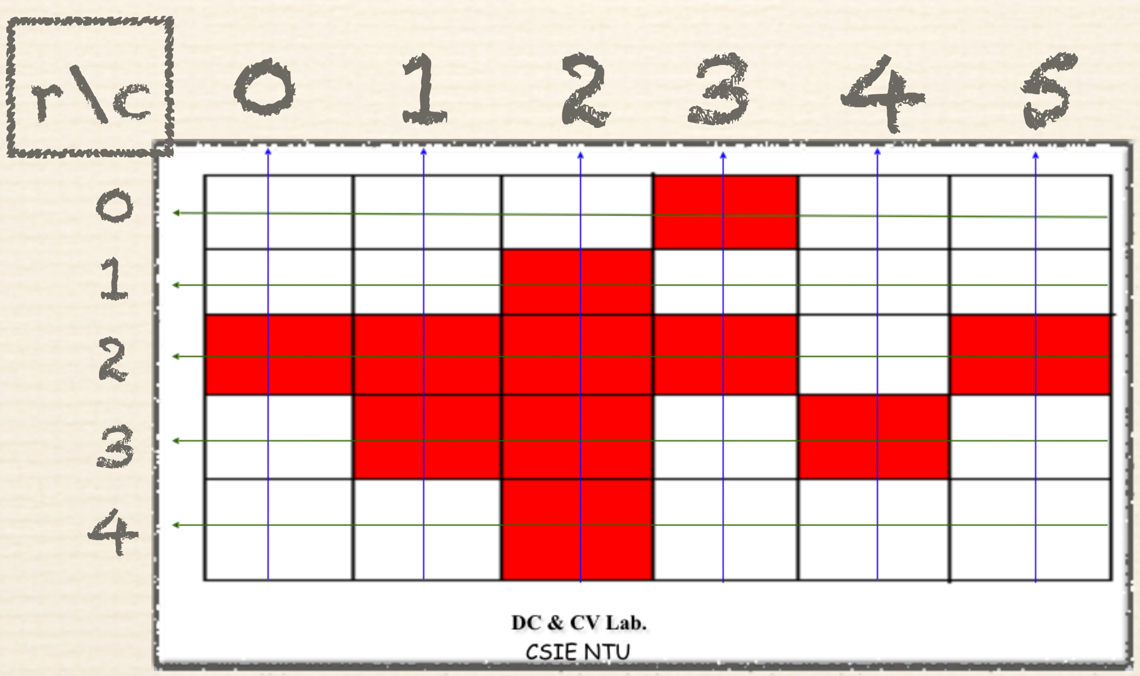
- diagonal projection from upper left to lower right

$$P_E(e) = \#\{(r,c) \in R | r - c = e\}$$

- Area

$$A = \sum_{(r,c)\in R} 1 = \sum_r \sum_{\{c|(r,c)\in R\}} 1 = \sum_r P_H(r)$$

So does column



3.3 Signature Properties (cont')

- rmin: top row of bounding rectangle

$$rmin = \min\{r|(r,c) \in R\} = \min\{r|P_H(r) \neq 0\}$$

- rmax; bottom row of bounding rectangle

$$rmax = \max\{r|(r,c) \in R\} = \max\{r|P_H(r) \neq 0\}$$

- cmin: leftmost column of bounding rectangle

$$cmin = \min\{c|(r,c) \in R\} = \min\{c|P_V(c) \neq 0\}$$

- cmax: rightmost column of bounding rectangle

$$cmax = \max\{c|(r,c) \in R\} = \max\{c|P_V(c) \neq 0\}$$

3.3 Signature Properties (cont')

- Area $$A = \sum_{(r,c)\in R} 1 = \sum_{r} \sum_{\{c|(r,c)\in R\}} 1 = \sum_{r} P_H(r)$$ So does column

- row centroid

$$\bar{r} = \frac{1}{A} \sum_{(r,c)\in R} r = \frac{1}{A} \sum_{r} \sum_{\{c|(r,c)\in R\}} r$$
$$= \frac{1}{A} \sum_{r} r \sum_{\{c|(r,c)\in R\}} 1 = \frac{1}{A} \sum_{r} r P_H(r)$$

- column centroid

$$\bar{c} = \frac{1}{A} \sum_{(r,c)\in R} c = \frac{1}{A} \sum_{c} \sum_{\{r|(r,c)\in R\}} c$$
$$= \frac{1}{A} \sum_{c} c \sum_{\{r|(r,c)\in R\}} 1 = \frac{1}{A} \sum_{c} c P_V(c)$$

- diagonal centroid

$$\bar{d} = \frac{1}{A} \sum_{d} d P_D(d) \qquad \bar{e} = \frac{1}{A} \sum_{e} e P_E(e)$$

3.3 Signature Properties (cont')

- second row moment from horizontal projection

$$\mu_{rr} = \frac{1}{A}\sum_{r}(r-\bar{r})^2 P_H(r)$$

- second column moment from vertical projection

$$\mu_{cc} = \frac{1}{A}\sum_{c}(c-\bar{c})^2 P_V(c)$$

- second diagonal moment

$$\mu_{dd} = \frac{1}{A}\sum_{d}(d-\bar{d})^2 P_D(d)$$

3.3 Signature Properties (cont')

$$\mu_{rr} = \frac{1}{A}\sum_{r}(r-\bar{r})^2 P_H(r)$$

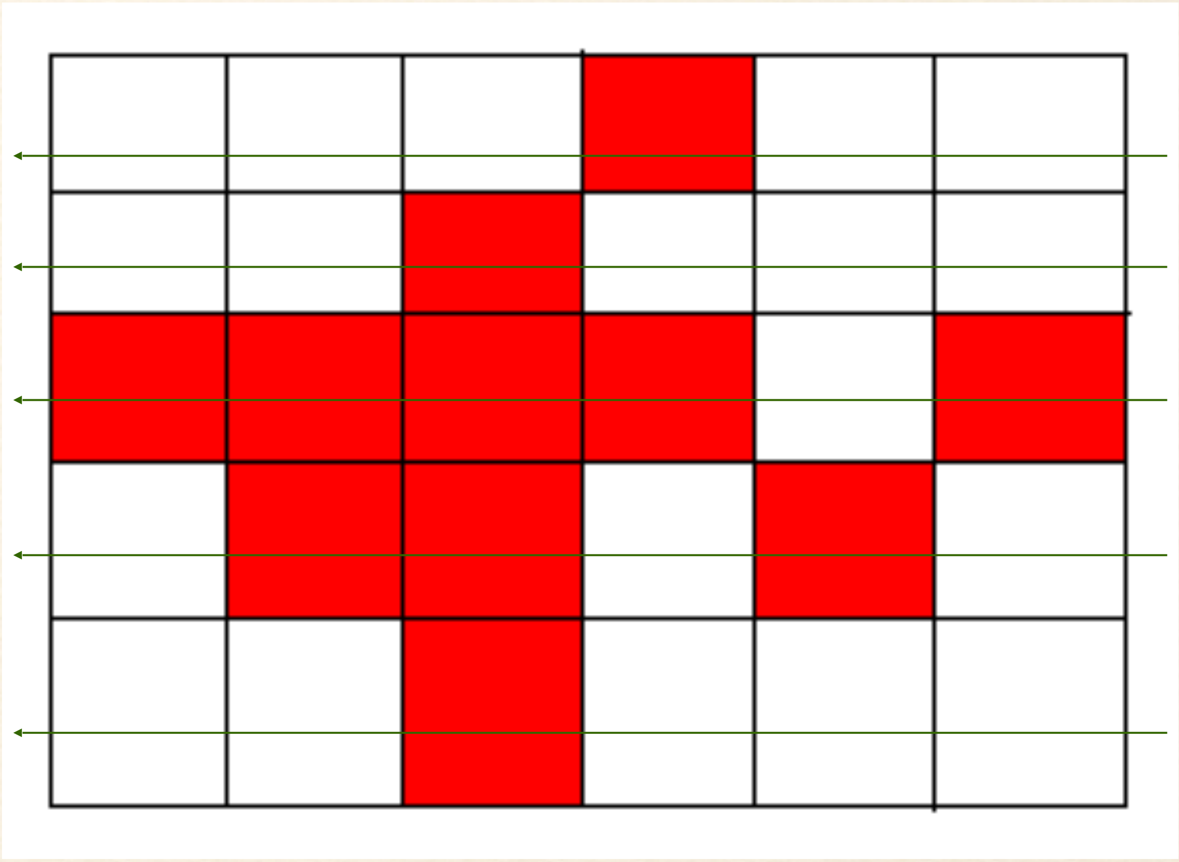
$$(0-\bar{r})^2 * P_H(0) = (0-\bar{r})^2 * 1$$

$$(1-\bar{r})^2 * P_H(1) = (1-\bar{r})^2 * 1$$

$$(2-\bar{r})^2 * P_H(2) = (2-\bar{r})^2 * 5$$

$$(3-\bar{r})^2 * P_H(3) = (3-\bar{r})^2 * 3$$

$$(4-\bar{r})^2 * P_H(4) = (4-\bar{r})^2 * 1$$



$$P_H(r) = \#\{c \mid (r,c) \in R\}$$

3.3 Signature Properties (cont')

- second diagonal moment related to $\mu_{rc}, \mu_{rr}, \mu_{cc}$

$$
\begin{aligned}
\mu_{dd} &= \frac{1}{A} \sum_{d} \sum_{\{(r,c)\in R \mid r+c=d\}} (r + c - \bar{r} - \bar{c})^2 \\
&= \frac{1}{A} \sum_{(r,c)\in R} [(r - \bar{r}) + (c - \bar{c})]^2 \\
&= \frac{1}{A} \sum_{(r,c)\in R} (r - \bar{r})^2 + 2(r - \bar{r})(c - \bar{c}) + (c - \bar{c})^2 \\
&= \mu_{rr} + 2\mu_{rc} + \mu_{cc}
\end{aligned}
$$

- second mixed moment can be obtained from projection

$$
\mu_{rc} = \frac{\mu_{dd} - \mu_{rr} - \mu_{cc}}{2}
$$

3.3 Signature Properties (cont')

- second diagonal moment related to $\mu_{rc}, \mu_{rr}, \mu_{cc}$

$$\mu_{ee} = \mu_{rr} - 2\mu_{rc} + \mu_{cc}$$

- second mixed moment can be obtained from projection

$$\boxed{\mu_{rc} = \frac{\mu_{rr} + \mu_{cc} - \mu_{ee}}{2}}$$

$$\boxed{\mu_{rc} = \frac{\mu_{dd} - \mu_{rr} - \mu_{cc}}{2}}$$

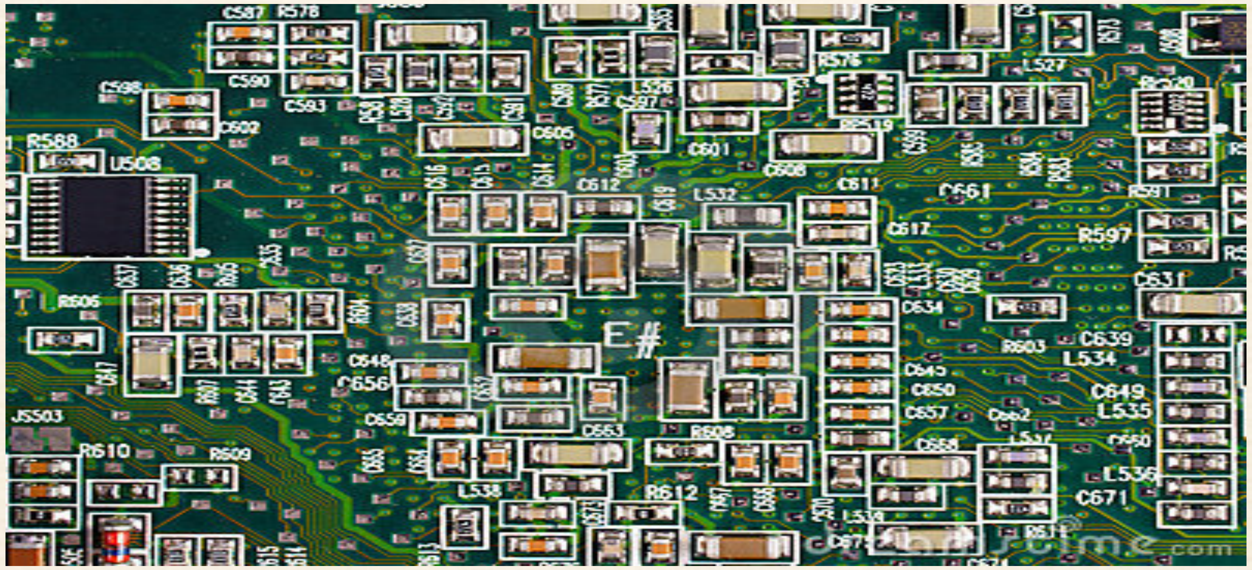
- mixed moment μ_{rc} obtained directly from μ_{dd} and μ_{ee}

$$\mu_{rc} = \frac{\mu_{dd} - \mu_{ee}}{4}$$

3.3.1 Signature Analysis to Determine the Center and Orientation of a Rectangle

- signature analysis: important because of easy, fast implementation

- surface mount device (SMD) inspection: position and orientation of parts



3.3.1 Signature Analysis to Determine the Center and Orientation of a Rectangle (cont')

- determine center $(\Delta x, \Delta y)$ of rectangle by corner location
- side lengths w, h orientation angle θ

clockwise

1. Rotation matrix : $$\begin{pmatrix} x_{rot} \\ y_{rot} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

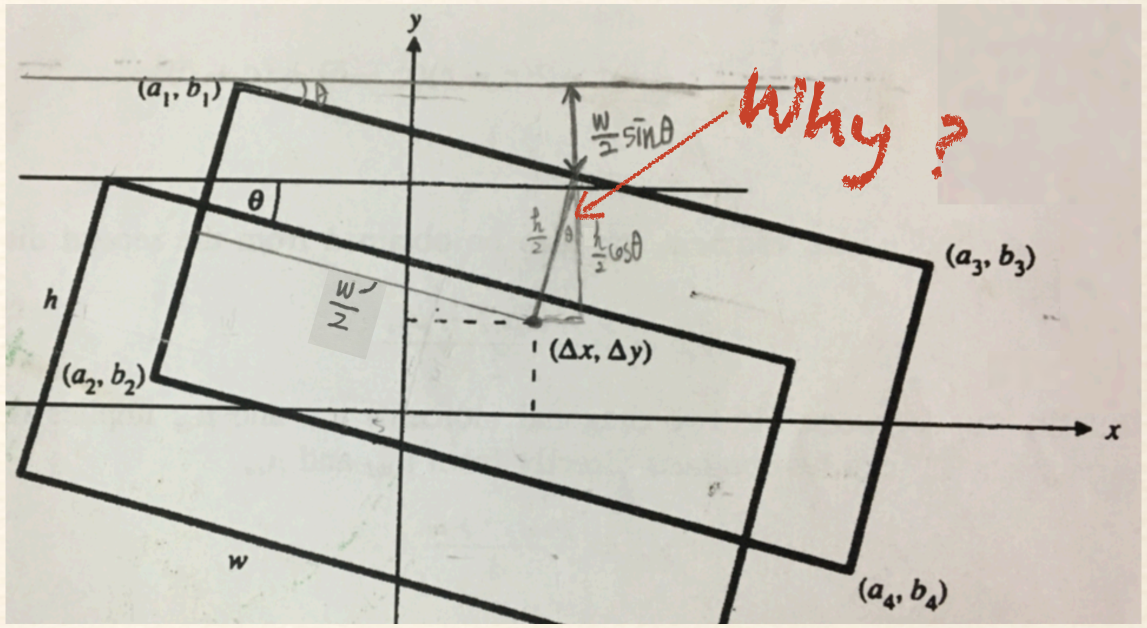
2. \+ Shift : $$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

3. coordinate $(\Delta x, \Delta y)$: $$\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = \begin{pmatrix} a_1 + \frac{w}{2}\cos\theta - \frac{h}{2}\sin\theta \\ b_1 - \frac{w}{2}\sin\theta - \frac{h}{2}\cos\theta \end{pmatrix}$$

Why ?

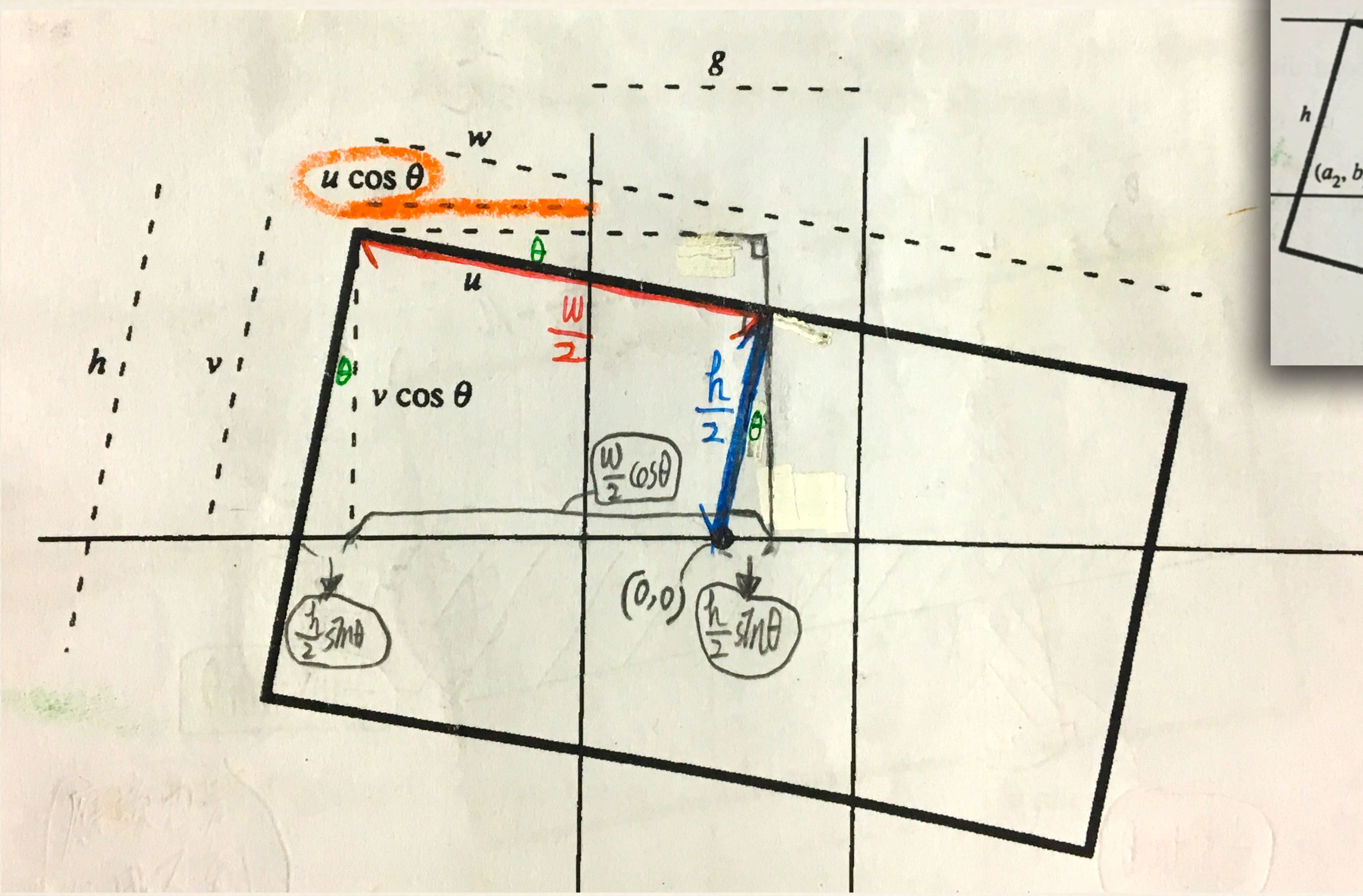
3.3.1 Signature Analysis to Determine the Center and Orientation of a Rectangle (cont')

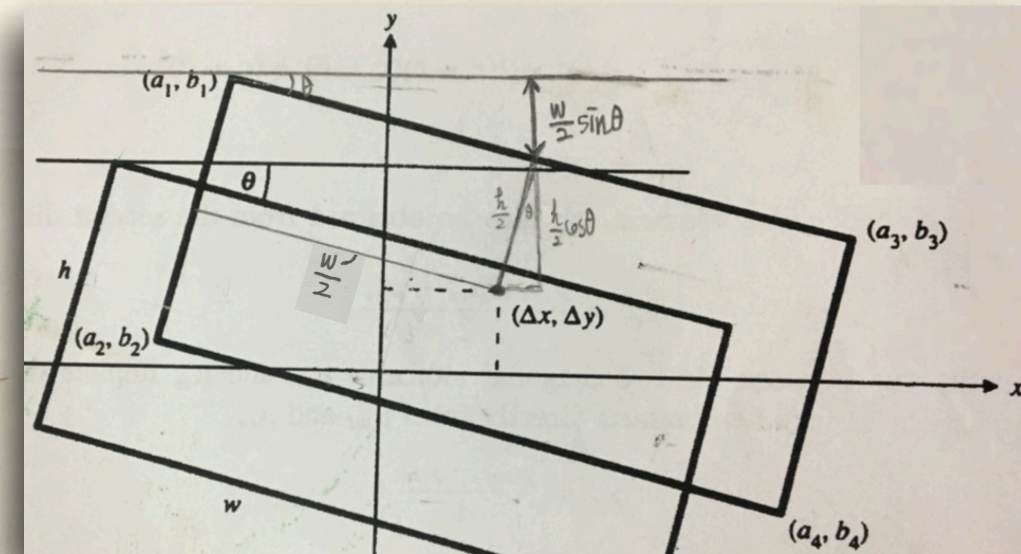
- geometry for determining the translation of the center of a rectangle
- in terms of :
  1. the location of one corner : (a_1,b_1)
  2. the length of its sides : w, h
  3. its orientation angle : θ



$$\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = \begin{pmatrix} a_1 + \frac{w}{2}\cos\theta - \frac{h}{2}\sin\theta \\ b_1 - \frac{w}{2}\sin\theta - \frac{h}{2}\cos\theta \end{pmatrix}$$

3.3.1 Signature Analysis to Determine the Center and Orientation of a Rectangle (cont')



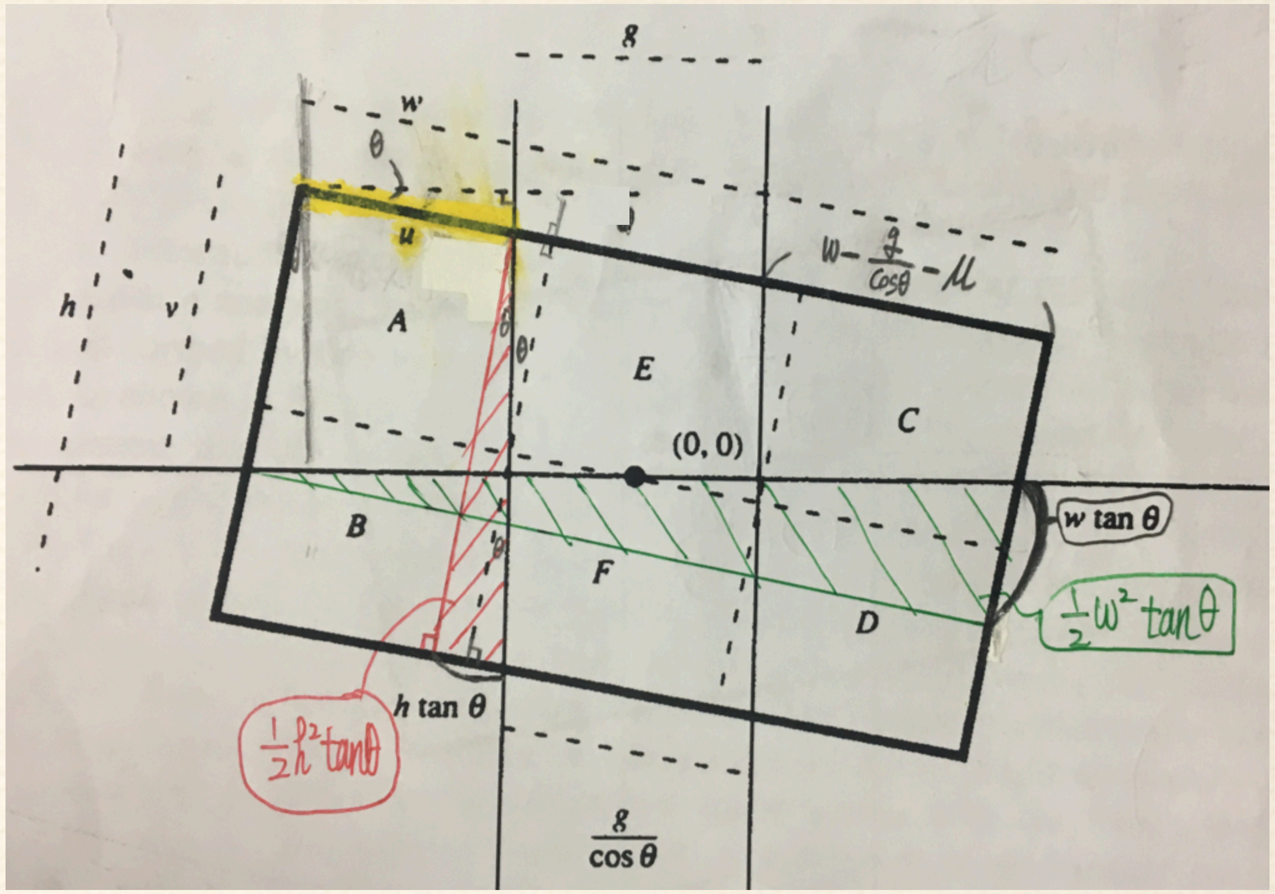


Upon substituting $-g/2 - u\cos\theta$ for a_1 and $v\cos\theta$ for b_1, we obtain

$$\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = \begin{pmatrix} \frac{-g}{2} - u\cos\theta + \frac{w}{2}\cos\theta - \frac{h}{2}\sin\theta \\ v\cos\theta - \frac{w}{2}\sin\theta - \frac{h}{2}\cos\theta \end{pmatrix}$$

3.3.1 Signature Analysis to Determine the Center and Orientation of a Rectangle (cont')

- Calculate each area after our rotation



$$\boxed{A + B = uh + \frac{1}{2}h^2 \tan\theta}$$

$$C + D = \left(w - \frac{g}{\cos\theta} - u\right)h - \frac{1}{2}h^2 \tan\theta$$

$$(C + D) - (A + B) = \left(w - \frac{g}{\cos\theta}\right)h - 2uh - h^2 \tan\theta$$

$$2uh = -(C + D) + (A + B) + \left(w - \frac{g}{\cos\theta}\right)h - h^2 \tan\theta$$

$$u = \frac{(A + B) - (C + D)}{2h} + \frac{1}{2}\left(w - \frac{g}{\cos\theta}\right) - \frac{1}{2}h \tan\theta$$

3.3.1 Signature Analysis to Determine the Center and Orientation of a Rectangle (cont')

- Calculate each area after our rotation

$$\boxed{A + E + C = vw - \frac{1}{2}w^2 \tan\theta}$$

$$B + F + D = (h - v)w + \frac{1}{2}w^2 \tan\theta$$

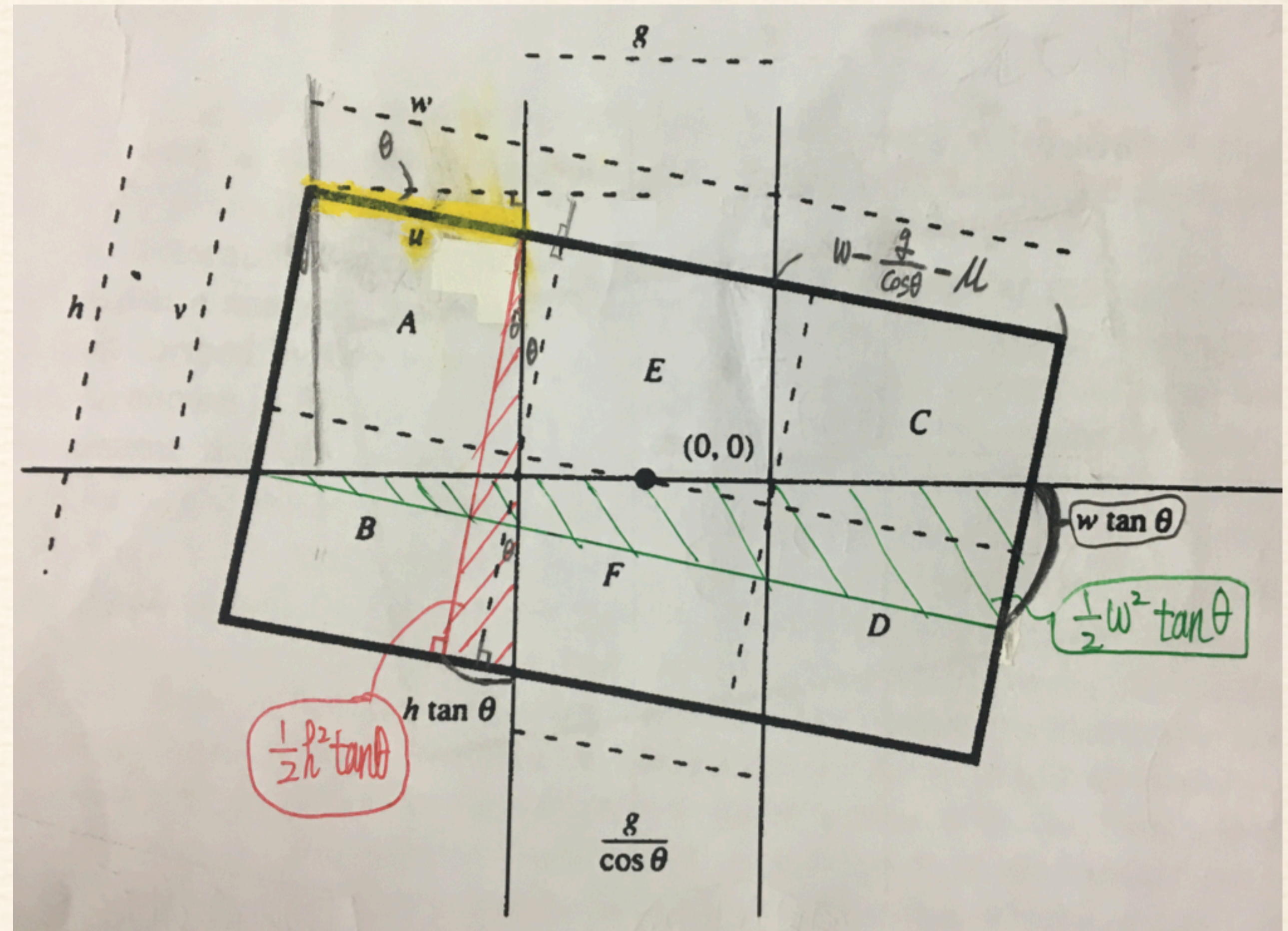


Figure 3.14 Geometry for determining the lengths u and v in terms of the measured areas A, B, C, D, E, and F.

$$\begin{aligned}
&(A + E + C) - (B + F + D) \\
&= vw - \frac{1}{2}w^2 \tan\theta - hw + vw - \frac{1}{2}\tan\theta \\
&= 2vw - hw - w^2 \tan\theta
\end{aligned}$$

$$2vw = (A + E + C) - (B + F + D) + hw + w^2 \tan\theta$$

$$v = \frac{(A + E + C) - (B + F + D)}{2w} + \frac{h}{2} + \frac{w}{2}\tan\theta$$

3.3.1 Signature Analysis to Determine the Center and Orientation of a Rectangle (cont')

$$\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = \begin{pmatrix} \frac{-g}{2} - u\cos\theta + \frac{w}{2}\cos\theta - \frac{h}{2}\sin\theta \\ v\cos\theta - \frac{w}{2}\sin\theta - \frac{h}{2}\cos\theta \end{pmatrix}$$

$$u = \frac{(A+B)-(C+D)}{2h} + \frac{1}{2}\left(w - \frac{g}{\cos\theta}\right) - \frac{1}{2}h\tan\theta$$

$$\Delta x = \frac{-g}{2} - u\cos\theta + \frac{w}{2}\cos\theta - \frac{h}{2}\sin\theta$$

$$\Delta x = \cancel{\frac{-g}{2}} - \left[\frac{(A+B)-(C+D)}{2h} + \frac{1}{2}\left(\cancel{w} - \cancel{\frac{g}{\cos\theta}}\right) - \cancel{\frac{1}{2}h\tan\theta}\right]\cos\theta$$

$$+ \cancel{\frac{w}{2}\cos\theta} - \cancel{\frac{h}{2}\sin\theta}$$

$$\Delta x = \frac{(C+D)-(A+B)}{2h}\cos\theta$$

$$v = \frac{(A+E+C)-(B+F+D)}{2w} + \frac{h}{2} + \frac{w}{2}\tan\theta$$

$$\begin{aligned}
\Delta y &= v\cos\theta - \frac{w}{2}\sin\theta - \frac{h}{2}\cos\theta \\
&= \left[\frac{(A+E+C)-(B+F+D)}{2w} + \frac{h}{2} + \frac{w}{2}\tan\theta\right]\cos\theta - \frac{w}{2}\sin\theta - \frac{h}{2}\cos\theta \\
&= \frac{(A+E+C)-(B+F+D)}{2w}\cos\theta + \frac{h}{2}\cos\theta + \frac{w}{2}\sin\theta - \frac{w}{2}\sin\theta - \frac{h}{2}\cos\theta \\
&= \frac{(A+E+C)-(B+F+D)}{2w}\cos\theta
\end{aligned}$$

3.3.1 Signature Analysis to Determine the Center and Orientation of a Rectangle (cont')

$$\Delta x = \frac{(C+D)-(A+B)}{2h}\cos\theta$$

$$\Delta y = \frac{(A+E+C)-(B+F+D)}{2w}\cos\theta$$

- where rotation angle

$$E+F = \frac{hg}{\cos\theta}$$

$$\cos\theta = \frac{hg}{E+F}$$

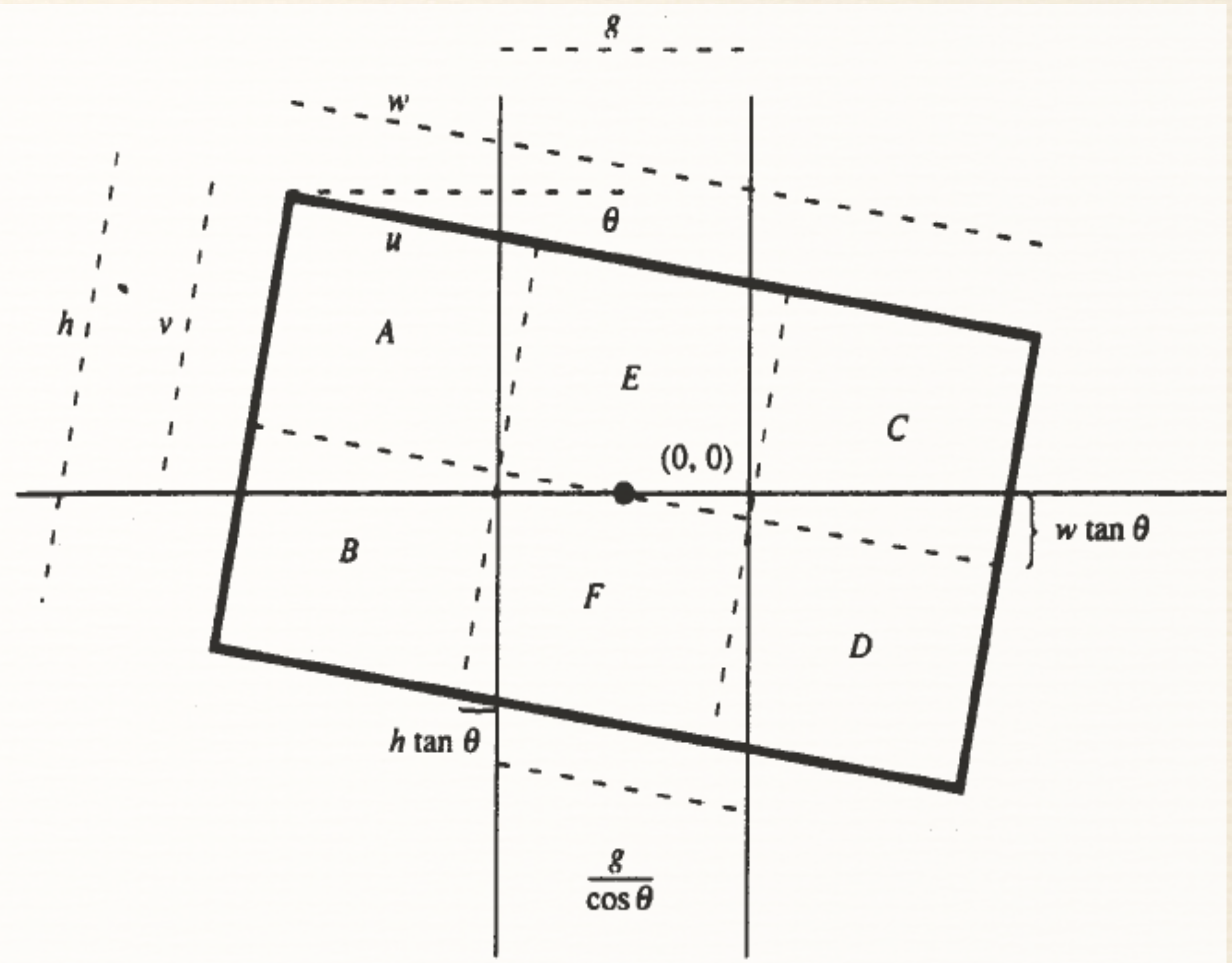
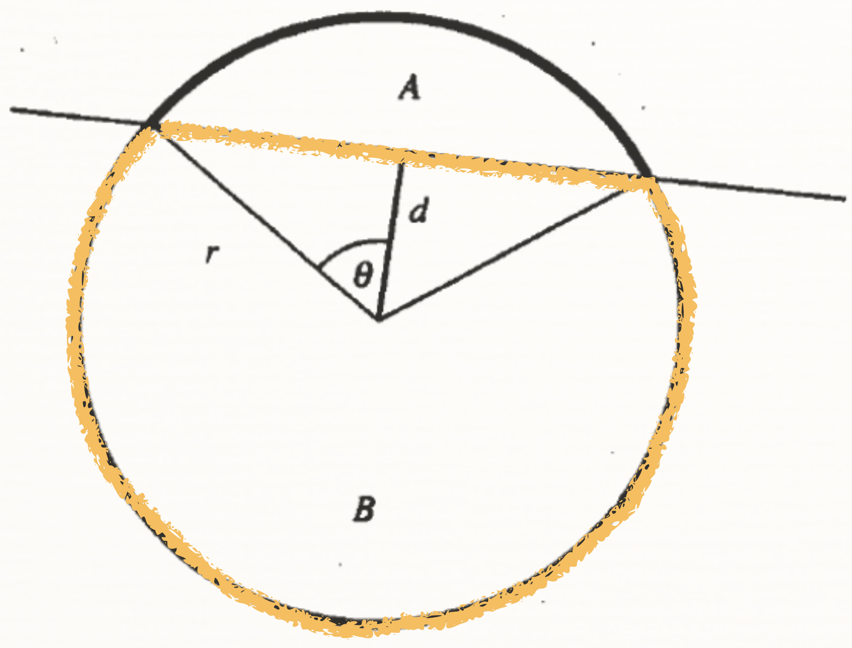


Figure 3.14 Geometry for determining the lengths u and v in terms of the measured areas A, B, C, D, E, and F.

3.3.2 Signature Analysis to Determine the Center and Orientation of a Circle

- partition the circle into four quadrants formed by two orthogonal lines which meet inside the circle

- geometry for the circle, its center, and a chord



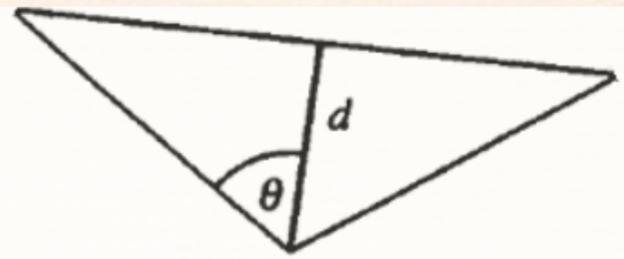
$$A + \boxed{B} = \pi r^2$$
$$r = \sqrt{\frac{A+B}{\pi}}$$
$$A = \frac{r^2}{2}[2\theta - \sin 2\theta]$$
$$\frac{2\pi A}{A+B} = 2\theta - \sin 2\theta$$
$$d = \sqrt{\frac{A+B}{\pi}} \cos\theta$$

Why ?

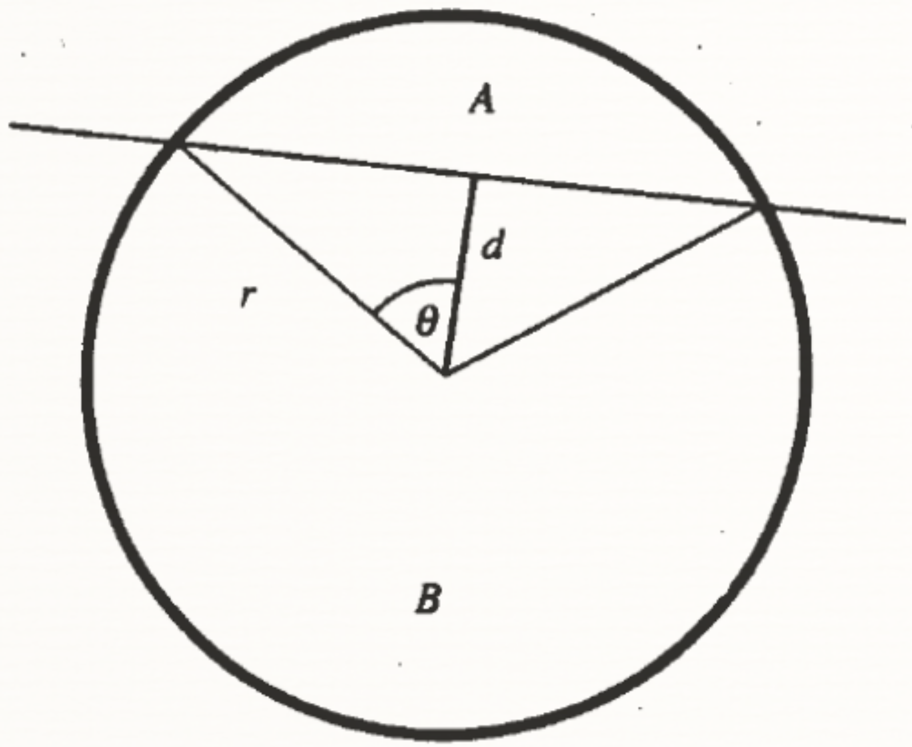
Figure 3.15 Geometry for the circle, its center, and a chord.

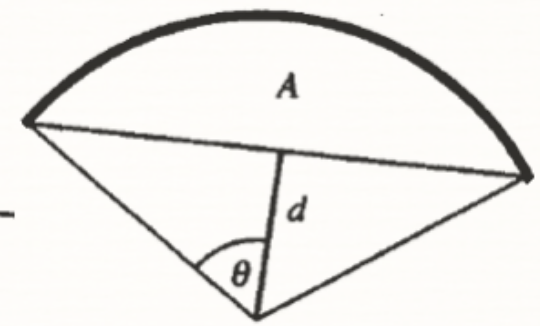
3.3.2 Signature Analysis to Determine the Center and Orientation of a Circle (cont')

$$\boxed{\theta = \cos^{-1}\frac{d}{r}}$$



$$= d\sqrt{r^2 - d^2}$$





$$= r^2\theta$$

$$A = r^2 \cos^{-1}\frac{d}{r} - d\sqrt{r^2 - d^2}$$

$$= r^2\left[\cos^{-1}\frac{d}{r} - \frac{d}{r}\sqrt{1 - \left(\frac{d}{r}\right)^2}\,\right]$$

$$= \frac{r^2}{2}[2\theta - \sin 2\theta]$$

Figure 3.15 Geometry for the circle, its center, and a chord.

由 $\begin{cases} \frac{d}{r} = \cos\theta \\ \sqrt{1-(\frac{d}{r})^2} = \sqrt{1-\cos^2\theta} = \sin\theta \end{cases}$

得 $2\frac{d}{r}\sqrt{1-(\frac{d}{r})^2} = 2\cos\theta\sin\theta = \sin 2\theta$

3.3.2 Signature Analysis to Determine the Center and Orientation of a Circle (cont')

- circle projected onto the four quadrants of the projection index image

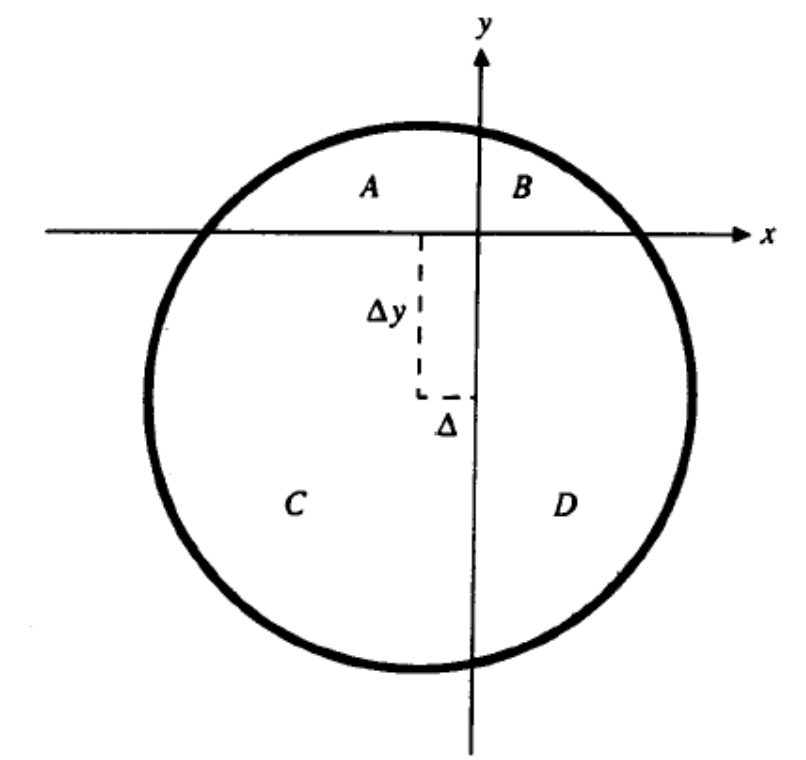


Figure 3.16 Circle projected onto the four quadrants of the projection index image.

3.3.2 Signature Analysis to Determine the Center and Orientation of a Circle (cont')

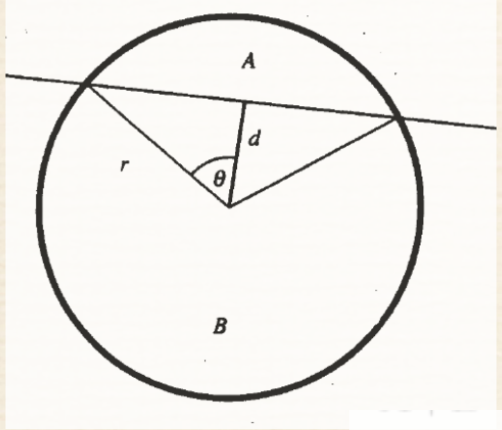
- each quadrant area from histogram of the masked projection
- Δy is positive if *A + B > C + D* otherwise it is negative

$$\boxed{A = \frac{r^2}{2}[2\theta - \sin 2\theta]}$$

Think of A+B as 2A
A+B+C+D=πr^2

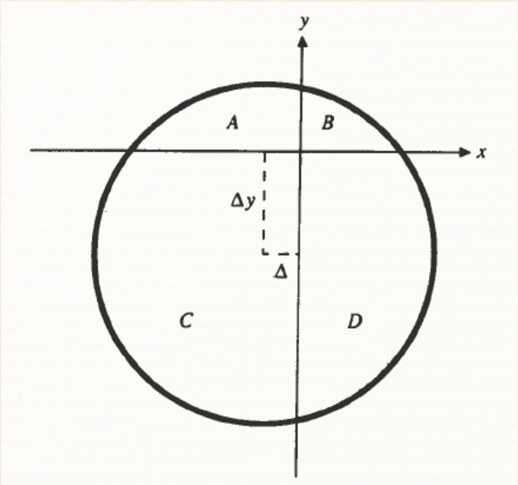
$$|\Delta y| = \sqrt{\frac{A+B+C+D}{\pi}}\cos\theta_y$$
$$\frac{2\pi(A+B)}{A+B+C+D} = 2\theta_y - \sin 2\theta_y$$

- Δx is positive if *B + D > A + C*, otherwise it is negative



$$d = \sqrt{\frac{A+B}{\pi}}\cos\theta$$

$$|\Delta x| = \sqrt{\frac{A+B+C+D}{\pi}}\cos\theta_x$$
$$\frac{2\pi(B+D)}{A+B+C+D} = 2\theta_x - \sin 2\theta_x$$



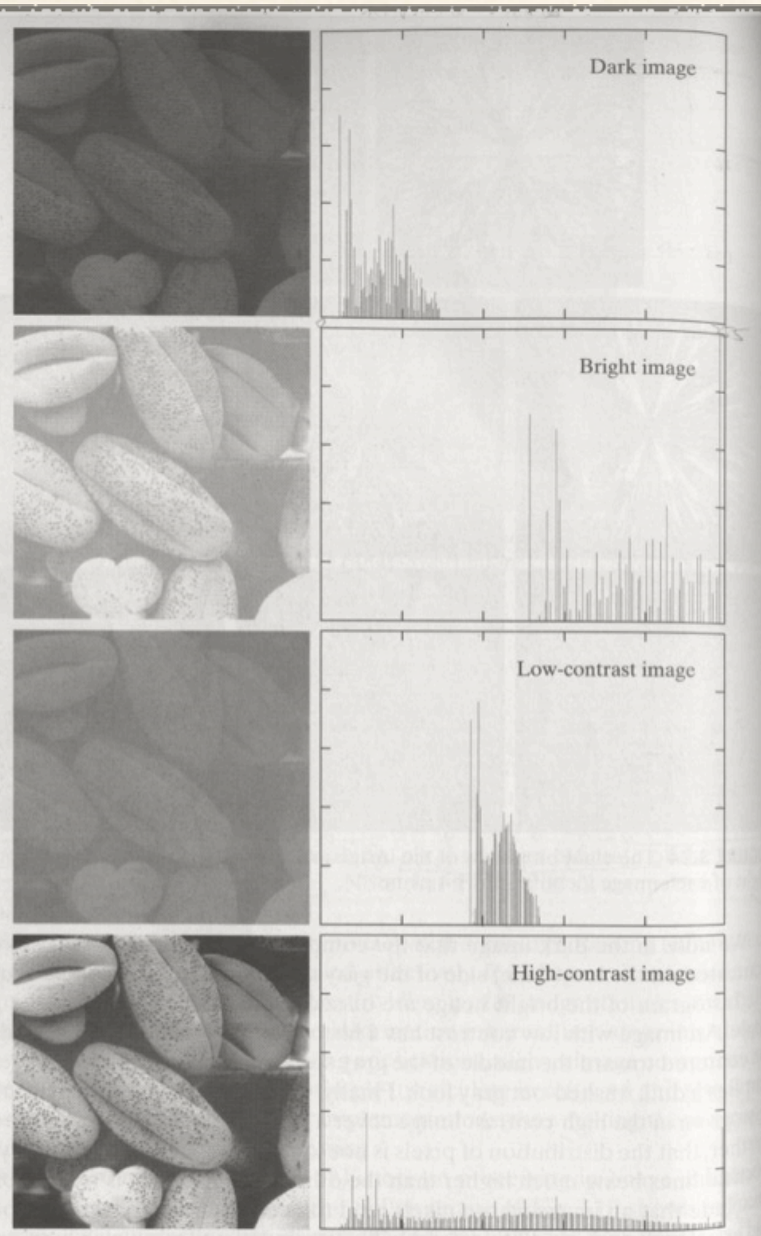
3.4 Summary

- region properties from
  - connected components
  - or signature analysis

Histogram Equalization (Homework)

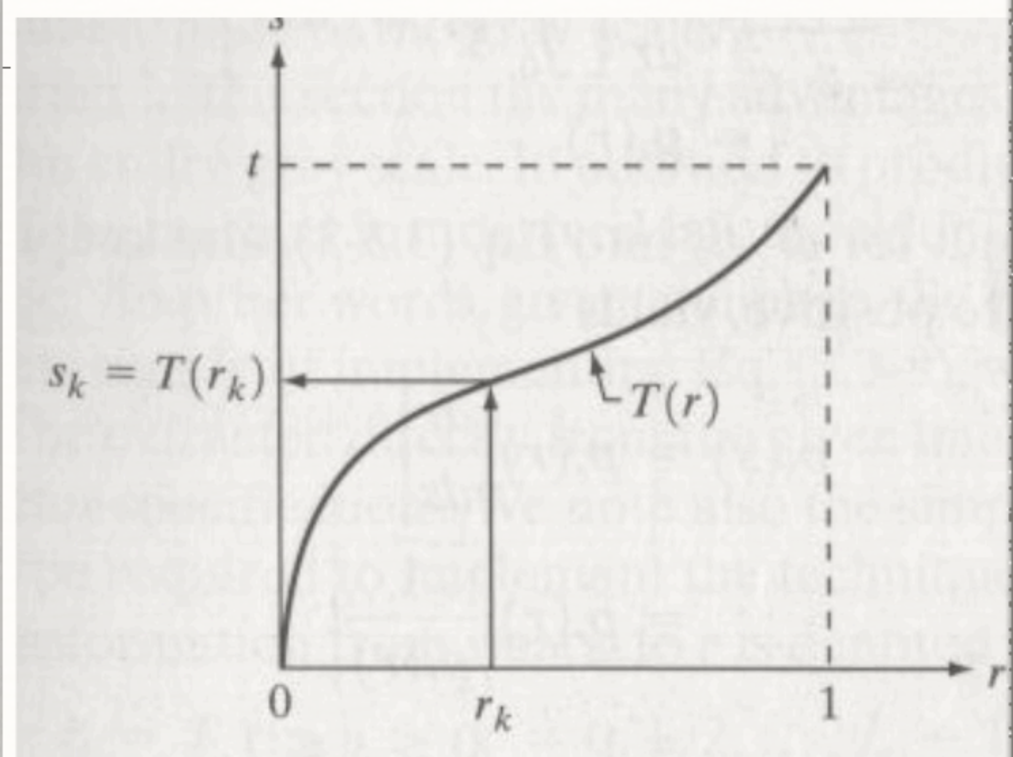
- a method in image processing of contrast adjustment using the image's histogram
- pixel transformation $s = T(r)$
- r, s: original, new intensity, T: transformation
- T(r) single-valued, monotonically increasing
- $0 \leq T(r) \leq 255$ for $0 \leq r \leq 255$

Histogram Equalization (Homework)



a b

FIGURE 3.15 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)



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Histogram Equalization (Homework)

- histogram equalization histogram linearization $$\boxed{s_k = 255 \sum_{j=0}^{k} \frac{n_j}{n}}$$
- $k = 0, 1, \ldots, 255,\ n_j$: number of pixels with intensity j
- n: total number of pixels
- for every pixel if $I(im, i, j) = k$ then $I(imhe, i, j) = s_k$

Histogram Equalization (Homework)

- Write a program to do histogram equalization
- Write a program to generate images and histograms :

  (a) original image and its histogram

  (b) image with intensity divided by 3 and its histogram

  (c) image after applying histogram equalization to (b) and its histogram

Histogram Equalization (Homework)

- Project due Oct. 8 :
- More information on our course page